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Wrench Analysis for 3-D Model Used in Robotic End-Effector

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In this paper, wrench analysis of a new proposed 3-D robotic model is discussed and applied. The model is basically used for calculating applied force through known spring stiffnesses and concerned compressive displacements. The wrench is correlated by already determined Jacobian matrix with global displacements. Local displacements are determined practically by applying load vertically in centrer. The global displacements (taken as reference) are calculated by congruence matrix through wrench analysis and shown. The theoretical relationship between global displacements and individual local displacements is also calculated and shown. Besides this practical determination of the wrench analysis is also verified by applying force on any leg of the model.

Key words. Wrench analysis, 3 - D robotic model.

Introduction

To calculate the wrench of any manufactured 3-D model is an extreme work rather than its twist. This is the model, which is used to determine its wrench analysis due to its turning affect from its three directions. An important elastic relationship is obtained of individual contact forces with externally applied global wrench. The wrench equation can be used to solve the forces in any statically indeterminate grasp problem. The significance of this relationship will be emphasized in the subsquent section (Kerr *et al* 1991).

To achieve the desired in-grasp manipulation, some preload has to be applied along some of the contacts in order to produce the effective global wrench 'w'. The present interest is to achieve a practicable solution such that a desired manipulation of objects can be achieved by preloading the minimum number of contacts with minimum possible preloads. It should be noted that the twist is in the axis coordinates that are its translational terms appear before rotational (Ghaffor and Kerr 1992).

δW is an infinitesimal wrench in ray coordinates in the form of δf, δm, and δd that is the infinitesimal twist of the grasped object in ray coordinates (Ghaffor *et al* 2000). The parameters of external infinitesimal wrench and body infinitesimal twist can be given by defining δf as force vector of (δfx, δFy, δFz) and δM as moment vector of (δMx, δMy, δMz) and δd as vector of translational displacements of (δ x, δy, δz).

A grasp with this stiffness matrix provides a restraint along six degrees of freedom when an object is subjected to an external wrench. The grasp arrangement does not have capability to induce fine motion in full dexterity and in particular cannot manipulate the object along z-axis. This can be visualized by substituting relationship of vector of preload magnitude δf and δw (Ghaffor *et al* 2000).

Any twist vector in the twist space under the mapping is a linear combination of these six twist vectors which correspond to be linearly independent. The six wrench vectors define a basis for the wrench space (Klafter *et al* 1989). Any wrench vector in this wrench space under mapping is a linear combination of these six wrench vectors.

Wrench analysis is investigated upon 3-D robotic model comprising of six legs attached with two different diameter platforms by spherical joints (Soomroza 2001). These joints give six degrees of freedom (three translatory and three rotationary) to calculate wrench matrix. A wrench is like torque having direction in x, y, z directions while wrench is the combination of force and moments in x, y, z directions.

Methodology. A 3-D model is used for grasping the object. This 3-D model is used in the end-Effector of the robotic manipulator. It possesses six legs joined by prismatic joints with inside springs as shown in Fig 1. These legs are fitted with two platforms giving six degrees of freedom. These motions are calculated by wrench analysis through applied forces 'F'stiffness [K] and compressive displacements as under.

Wrench Analysis. Since, we know that F=[K]. δd multiplying both sides by [J] we get:

$[J].F = [J].[K].\ \delta d$

or $W = [J].[K].\ \delta d$

But $\delta d = [J^t].\ \delta d$

putting this value, we get

$W = [J]\cdot[K]\cdot [J^t]\cdot \delta D$

$\therefore\ W = [K_g]\cdot \delta\, D$

or $\delta D = [K_g]^{-1}.W$

δD or δd then can be compared with that of practical obtained.

Where;

$[J]$ = Jacobian Matrix

F = Applied Force

$[K]$ = Stiffness Matrix

δd = Local displacement

$\delta\, D$ = Global displacement

$[J^T]$ = Transpose of Jacobian

W = Wrench Analysis

$[K_g]$ = Congruence Matrix

From above equation we get:

i.e. $\delta w = J_g\, [k]\, J^T.\ \delta D_g$

where;

$$\delta w = \begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ Mz \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.00 \\ -9.81 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}$$

and $J_g.[k]J^{T}$'. is called congruence transformation denoted by (K_g).

$$\begin{bmatrix} 0.00 \\ 0.00 \\ -9.81 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix} = \begin{bmatrix} 0.4330 & 0.0000 & -0.5000 & -0.4330 & -0.0000 & 0.5000 \\ 0.2500 & 0.5000 & 0.0000 & -0.2500 & -0.5000 & -0.0000 \\ 0.8660 & 0.8660 & 0.8660 & 0.8660 & 0.8660 & 0.8660 \\ -48.1180 & -96.2371 & -48.1180 & 48.1180 & 96.2371 & 48.1180 \\ 83.3430 & 0.0000 & -83.3430 & -83.3430 & 0.0000 & 83.3430 \\ 0.0000 & 0.0000 & -27.781 & 0.0000 & 0.0000 & -27.781 \end{bmatrix} \begin{bmatrix} 1.4098 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.4677 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.4335 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 3.6021 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.3278 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.3328 \end{bmatrix}$$

$$\begin{bmatrix} 0.4330 & 0.2500 & 0.8660 & -48.1185 & 83.3438 & 0.0000 \\ 0.0000 & 0.5000 & 0.8660 & -96.2371 & 00.0000 & 0.0000 \\ -0.5000 & 0.0000 & 0.8660 & -48.1185 & -83.3438 & -27.7810 \\ -0.4330 & -0.2500 & 0.8660 & 48.1185 & -83.3438 & 0.0000 \\ -0.0000 & -0.5000 & 0.8660 & 96.2371 & 00.0000 & 0.0000 \\ 0.5000 & -0.0000 & 0.8660 & 48.1185 & 83.3438 & -27.7810 \end{bmatrix} \begin{bmatrix} \delta D_{g1} \\ \delta D_{g2} \\ \delta D_{g3} \\ \delta D_{g4} \\ \delta D_{g5} \\ \delta D_{g6} \end{bmatrix}$$

$$\begin{bmatrix} 0.00 \\ 0.00 \\ -9.81 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix} = 1.0e{+}004\text{x} \begin{bmatrix} 0.0002 & 0.0001 & -0.0001 & -0.0038 & 0.0296 & 0.0001 \\ 0.0001 & 0.0001 & -0.0000 & -0.0195 & 0.0104 & -0.0000 \\ -0.0001 & -0.0000 & 0.0008 & 0.0075 & -0.0166 & -0.0067 \\ -0.0038 & -0.0195 & 0.0075 & 4.3900 & -0.9006 & 0.0135 \\ 0.0296 & 0.0104 & -0.0166 & -0.9006 & 5.4029 & 0.0233 \\ 0.0001 & -0.0000 & -0.0067 & 0.0135 & 0.0233 & 0.2135 \end{bmatrix} \begin{bmatrix} \delta D_{g1} \\ \delta D_{g2} \\ \delta D_{g3} \\ \delta D_{g4} \\ \delta D_{g5} \\ \delta D_{g6} \end{bmatrix}$$

$$\begin{bmatrix} \delta D_{g1} \\ \delta D_{g2} \\ \delta D_{g3} \\ \delta D_{g4} \\ \delta D_{g5} \\ \delta D_{g6} \end{bmatrix} = 1.0e{+}004\text{x} \begin{bmatrix} 0.0002 & 0.0001 & -0.0001 & -0.0038 & 0.0296 & 0.0001 \\ 0.0001 & 0.0001 & -0.0000 & -0.0195 & 0.0104 & -0.0000 \\ -0.0001 & -0.0000 & 0.0008 & 0.0075 & -0.0166 & -0.0067 \\ -0.0038 & -0.0195 & 0.0075 & 4.3900 & -0.9006 & 0.0135 \\ 0.0296 & 0.0104 & -0.0166 & -0.9006 & 5.4029 & 0.0233 \\ 0.0001 & -0.0000 & -0.0067 & 0.0135 & 0.0233 & 0.2135 \end{bmatrix} \begin{bmatrix} 0.00 \\ 0.00 \\ -9.81 \\ 0.00 \\ 0.00 \\ 0.00 \end{bmatrix}$$

$$\begin{bmatrix} \delta D_{g1} \\ \delta D_{g2} \\ \delta D_{g3} \\ \delta D_{g4} \\ \delta D_{g5} \\ \delta D_{g6} \end{bmatrix} = \begin{bmatrix} -0.6131 \\ 3.3722 \\ -1.9119 \\ 0.0165 \\ -0.0048 \\ -0.0596 \end{bmatrix}$$

But we also know from equation that $\delta d_1 = [J^T_g].\ \delta D_g$

$$\begin{bmatrix} \delta d_{11} \\ \delta d_{12} \\ \delta d_{13} \\ \delta d_{14} \\ \delta d_{15} \\ \delta d_{16} \end{bmatrix} \begin{bmatrix} 0.4330 & 0.2500 & 0.8660 & -48.1185 & 83.3438 & 0.0000 \\ 0.0000 & 0.5000 & 0.8660 & -96.2371 & 00.0000 & 0.0000 \\ -0.5000 & 0.0000 & 0.8660 & -48.1185 & -83.3438 & -27.7812 \\ -0.4330 & -0.2500 & 0.8660 & 48.1185 & -83.3438 & 0.0000 \\ -0.0000 & -0.5000 & 0.8660 & 96.2371 & 00.0000 & 0.0000 \\ 0.5000 & -0.0000 & 0.8660 & 48.1185 & 83.3438 & -27.7812 \end{bmatrix} \begin{bmatrix} -0.6131 \\ 3.0656 \\ -1.9119 \\ 0.0251 \\ -0.0144 \\ -0.0948 \end{bmatrix} \begin{bmatrix} -2.2697 \\ -1.5543 \\ -0.0865 \\ -1.0418 \\ -1.7572 \\ 0.0865 \end{bmatrix} \text{mm}$$

Putting these values of 'δD_g' on applying load of 9.81 N centrally in above equation

i.e. $w = [K_g] \cdot \delta D_g$

$$\begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ Mz \end{bmatrix} = 1.0e + 004 \times \begin{bmatrix} 0.0002 & 0.0001 & -0.0001 & -0.0038 & 0.0296 & 0.0001 \\ 0.0001 & 0.0001 & -0.0000 & -0.0195 & 0.0104 & -0.0000 \\ -0.0001 & -0.0000 & 0.0008 & 0.0075 & -0.0166 & -0.0067 \\ -0.0038 & -0.0195 & 0.0075 & 4.3900 & -0.9006 & 0.0135 \\ 0.0296 & 0.0104 & -0.0166 & -0.9006 & 5.4029 & 0.0233 \\ 0.0001 & -0.0000 & -0.0067 & 0.0135 & 0.0233 & 0.2135 \end{bmatrix}$$

$$\begin{bmatrix} -0.6131 \\ 3.3722 \\ -1.9119 \\ 0.0165 \\ -0.0048 \\ -0.0596 \end{bmatrix} = \begin{bmatrix} 0.3591 \\ 0.1642 \\ -10.0248 \\ -20.0884 \\ 66.0294 \\ 0.2422 \end{bmatrix} \text{ in 'N' for forces and N - mm for moments}$$

Practical value of Wrench (δW). We can also determine the values of wrench practically by applying on un-known force.

Suppose, we are applying un-known force to any leg of the model. On doing this action, we observe some observations as 2.75 mm on scale as local co-ordinate (δd_{LI}).

For finding the first value of force (F_1), we use the following formula:

$F_1 = [k] \cdot \delta d_{L1}$

$F_1 = [1.4098] \cdot (2.75) = 3.877$ N

Put above value in equation

i.e.

$\delta W = [J_g] \cdot [F]$

$$\begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ Mz \end{bmatrix} = \begin{bmatrix} 0.4330 & 0.0000 & -0.5000 & -0.4330 & -0.0000 & 0.5000 \\ 0.2500 & 0.5000 & 0.0000 & -0.2500 & -0.5000 & -0.000 \\ 0.8660 & 0.8660 & 0.8660 & 0.8660 & 0.8660 & 0.8660 \\ -48.1180 & -96.2370 & -48.1180 & -48.1180 & 96.2371 & 48.118 \\ 83.3430 & 0.0000 & -83.3430 & -83.3430 & 00.0000 & 83.343 \\ 0.0000 & 0.0000 & -27.7810 & 0.0000 & 00.0000 & -27.781 \end{bmatrix} \begin{bmatrix} 3.877 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{bmatrix}$$

$$\begin{bmatrix} Fx \\ Fy \\ Fz \\ Mx \\ My \\ Mz \end{bmatrix} = \begin{bmatrix} 0.1711 \\ 0.0988 \\ 0.3422 \\ -19.0160 \\ 32.9367 \\ 0.0000 \end{bmatrix} \text{ N for forces and N - mm for moments}$$

From above are required values of Force (Fx, Fy, Fz) and Torque (Mx, My, Mz) in shape of wrench (δw). We have also considered the value of stiffness (K_{av1}) Leg (1) as 1.4098 N/mm.

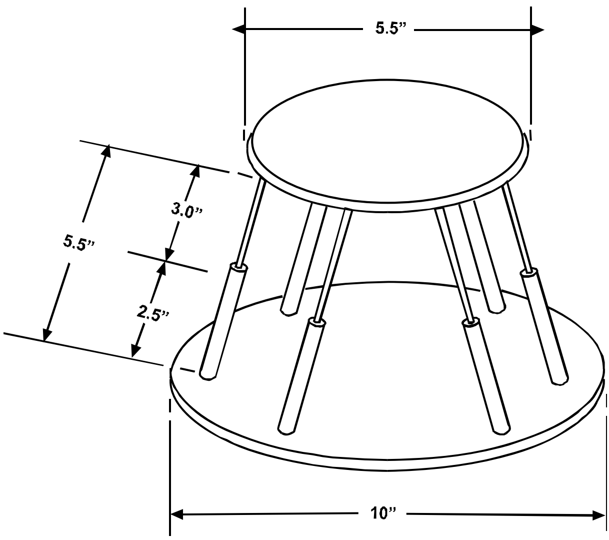

Fig 1. Front view of proposed 3-D Robotic Model.

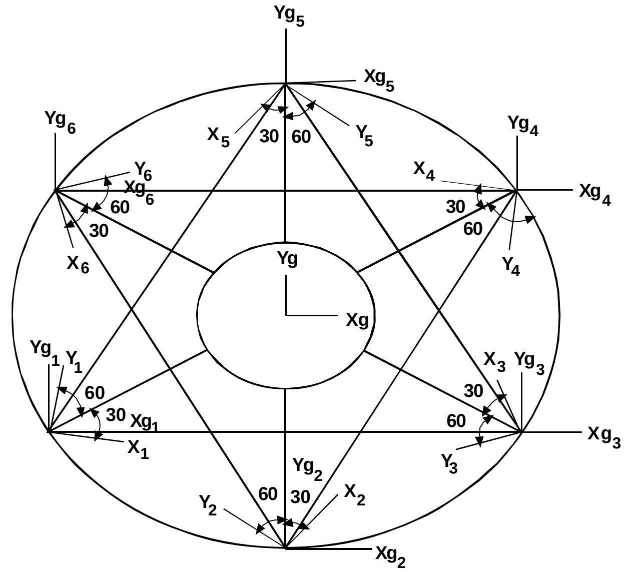

Fig 2. Correlation of Local and Global Displacements.

Results and Discussion

The structural design of the model is described briefly as above is shown in Fig 1. In Fig 2, brief transformation of local displacements into global displacements is shown. These local and global displacements are correlated and calculated by using wrench analysis in above sections. However, load was applied vertically and centrally downward. Parallel to z-axis initially. Thus, local displacements were determined experimentally as in Table 1. Then global displacements are calculated by using the relationship through wrench analysis as in Table 2. Thus, theoretical results of local displacements from global displacement by congruence matrix of wrench analysis are achieved. A practical wrench analysis in shape of wrench matrix has been calculated practically in above sections. Also, wrench

Table 1
Experimental readings

S.No.	Load applied (W) Centrally in Newton (N)	Displacement (δd) measured along each leg in mm δd_1	δd_2	δd_3	δd_4	δd_5	δd_6
1	(0, 0, - 9.81, 0, 0, 0)	- 1.50	- 1.50	- 1.30	- 1.40	- 1.30	-1.20
2	(0, 0, - 19.62, 0, 0, 0)	- 1.74	- 1.81	- 1.53	- 1.72	- 1.69	-1.65
3	(0, 0, - 29.43, 0, 0, 0)	- 1.91	- 1.89	- 1.82	- 1.79	- 1.75	-1.73

Table 2
Theoretical results

S.No.	Load applied (W) centrally in Newton (N)	Global displacement (δD) determined along each leg in mm (using relation $\delta D = [Kg]^{-1} \cdot W$)	Local displacement (δd) calculated along each leg in mm (using equation $\delta d = [J^t] . \delta D$)
1	(0, 0, - 9. 81, 0, 0, 0)	(- 0.6131, 3.3722, -1.9119) 0.0165, - 0.0048, - 0.0596)	(- 2.2697, - 1.5543, - 0.0865 - 1.0418, - 1.7572, 0.0865)
2	(0, 0, - 19.62, 0, 0, 0)	(- 1.2263, 6.7444, - 3.8238 0.0329, - 0.0096, - 0.1192)	(- 4.5395, - 3.1086, - 0.1731 - 2.0836, - 3.5145, 0.1731)
3	(0, 0, - 29.43, 0, 0, 0)	(- 4.5395, - 3.1086, - 0.1731) - 2.0836, - 3.5145, 0.1731)	(- 6.8092, - 4.6629, - 0.2596) - 3.1254, - 5.2717, 0.2596)

matrix depending upon elements of forces and moments in x,y,z directions have also been determined in above sections. In this way global displacements are converted into local displacements by already calculated Jacobian matrix in Table 2.

In matrix from, wrench analysis is shown by six elements. First three elements represent force analysis in X, Y and Z directions. While last three elements of matrix represent the moments in concerned directions.

Conclusion

The wrench analysis is applied and calculated on a new 3-D model shown in Fig 1. It can be fitted into robotic End-Effector. For this, concerned wrench equations are discussed. The relative factors are calculated and investigated practically as well as theoretically. Thus, unknown applied forces and concerned moments in x, y, and z-axes are calculated. The global displacements are determined by congruence and wrench analysis. Then local displacements are obtained by correlation of global displacements through jacobian (J). Besides wrench analysis, this 3-D Robotic model is also used for calculating the force and local or global displacements apart from its stiffness.

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