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Impact of Subsynchronous Resonance on System Dynamic Stability

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Experimental and theoretical studies of subsynchronous resonance phenomena are carried out on a power system composed of a small rating motor-generator set connected to an infinite bus by a series capacitor-compensated transmission line. Complete representation of the electromechanical system is adopted. The eigenvalue method of analysis is used to study the interaction between mechanical and electrical network dynamic behaviour to identify the various conditions under which the system would be subjected to torsional interaction. The impact of subsynchronous resonance on dynamic stability is also explored.

Key words: Subsynchronous resonance, Power systems, Dynamic stability.

Introduction

Subsynchronous resonance (SSR) continues to be a subject of concern and study by most of world utilities since it usually exists in system and its occurrence causes a severe damage to turbine-generator shaft assembly. The mechanical structure of large size generator consists of multi-stage turbines, generator rotor and exciter. The number of turbine stages depend on the size of the generating unit.

Because of some mechanical considerations in the design of the shaft and masses connected to the shaft, the mechanical shaft has a number of natural frequencies most of them below the synchronous frequency termed as subsynchronous frequencies. On the other side in the electrical system, there are a large number of natural frequencies. The number and values of these natural frequencies depend on the number and topology of circuit configuration that can be made by switching. For long time three phases short circuits at the machine terminals were considered the more severe disturbance on the mechanical structure of turbo-generators. Recently there has been increasing recognition of some other system disturbances that might have a harmful effect on the mechanical structure of large size turbo-generators. These disturbances are:

(i) Transmission line switchings. (ii) High speed reclosing of circuit breakers after fault clearing on line leaving power stations. (iii) Single pole operation of circuit breakers that produces alternating torque at twice the rated frequency. This torque affects the blades of the turbines. (iv) Subsynchronous resonance in series capacitor compensated system. (v) Faults at generator terminals out of phase synchronization. (vi) Full load trips.

Any one of these disturbances will cause the interchange of energies between elements of power system (Anderson 1979; Trudnowski and Dagle 1997). The objective of this paper is to contribute to the understanding of subsynchronous oscillations in series compensated transmission circuits of small rating motor-generator set connected to an infinite bus. Moreover an experimental oriented study is made to show the impact of subsynchronous oscillations applied to the small set of synchronous machine connected to an infinite bus through compensated line. To the best of the author's knowledge, no one has ever conducted an experimental study of this phenomenon. (IEEE Committee Reports 1976; 1979; 1985; Ronald *et al* 1995; Iravani and Edris 1995). This experimental realizaion work is followed by a theoretical verification using rigorous mathematical modelling and computer simulation.

Experimental Work. The effect of the presence of series capacitor compensation on the system dynamics has been realized experimentally. For this purpose an experimental set-up is arranged in the Laboratory (Figure 1). The system under investigation consists of: 1 KVA/.8 kW, 380 V, 1.6 A, 60 Hz, 4 pole synchronous generator driven by a D.C separately excited motor as a prime mover located at machine laboratory, King Saud University. The mechanical and electrical data of the generator and its driving shaft have been measured using conventional techniques (Table 1). The corresponding parameters are given in per unit.

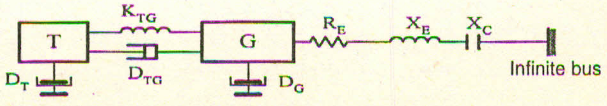

Fig 1. One line diagram of the experimental set-up.

Table 1
Electrical and mechanical data measured for the motor-generator set in p.u.

r_{kd}	= 0.056	X'_d	= .0784
r_{kq}	= 0.15	X_{ad}	= .729
X_{kd}	= 0.194	X_a	= .0549
X_d	= 0.78	r_{fd}	= .00275
X_{kq}	= 0.5	X_c	= 34.5/137.5 = .25
X_{ad}	= 0.706	X_E	= .274 p.u
X_{aq}	= 0.549	D_{tg}	= 0.5
X_q	= 0.627	D_{tt}	= 0.5
r_s	= 0.07	Q_t	= 101.0
X_c	= variable	Q_g	= 101.0
X'_q	$= X_q'' = .316$	K_{tg}	= 35.4
X_{kkq}	= 1.049	D_{gg}	= 0.2
X_{kkd}	= .923	V	= 1.00
X_{ffd}	= .73	P	= 1.00
X_f	= .0275	Q	= .25
		T	= .03

System Dynamic Response. The synchronous generator is connected to an infinite bus through a series capacitor compensated tie-line. The value of the capacitor is varied to investigate the effect of the degree of compensation on subsynchronous resonance oscillations. The generator speed deviation is recorded for each degree of compensation. It was found that the severity of the resulting oscillation is maximum for the case of Xc equal to 34.64 or 35 ohm with XL equal to 37.68 Ω, for one operating condition, as shown in (Table 2). This corresponds to a natural frequencyy of electrical oscillations equal to 16.667 Hz.

Table 3 shows again that in the case of two machines connected in parallel to a common bus and then tied to an infinite bus through a series compensated line, the severity of oscillation occurs when X_c is 29 ohm and X_L equal to 37.68 ohm. This correponds to natural frequency of electrical oscillation of 17.7 Hz. This value Xc is almost closed to the value causing sever oscillation one machine system.

This could be expected since the two identical machines operating in parallel have one machine equivalent. Meanwhile

Table 2
One machine variable connected to infinite bus R = 0 and $X_L = 37.68\ \Omega$

System variables						Observation
X_c	V_{inf}	Frequency	P_{inf}	Q_{inf}	$p.f_{inf}$	
29.1Ω	380	7.14	1174	1198	0.7 lagging	Operating smoothly no stress
34.64	380	16.667	1375.6	452.0	.95 lagging	Severe ocillation
35	380	16.667	1496.5	307	.98 lagging	Severe oscillation
36	380	-	1677	340	.98 lagging	Shocking oscillation less than the case of Xc = 35.
47.22	-	-	-	-	-	Machine shocking strongly can't take reading

Table 3
Two machine variables connected to infinite bus R = 0, XL = 37.68 Ω

System variables						Observation
X_c	V_{inf}	Frequency	P_{inf}	Q_{inf}	$p.f_{inf}$	
17.17	380	-	66	91.6	.1 lagging	Operating smoothly
23.9	380	-	542.6	136	.97 lagging	Beginning of torsional oscillation, machine operate under stress
29.1	380	17.7	3234	14.74	.91 lagging	Machine oscillates strongly and shocking
31.8	380	-	-	-	-	Sever torsional oscillation

the natural frequency of electrical oscillation depends on the transmission line parameters as well as the machine parameters.

Torsional Resonance Interaction. Usually the turbo generator shaft has many torsional modes. For each mode there is a natural frequency of the torsional resonance. The detailed mechanism by which electromechanical damping occurs is not yet understood, although the condition at which the instability situation occurs has been identified (El - Abiad 1983).

It is understood that, this condition occurs when the series resonance frequency of the L-C ω_n is approximated by $\omega_0 - \omega_m$ where ω_m is any one of the torsional resonance frequencies and ω_0 is the rotor synchronous speed. It has been understood that at this frequency the electrical system will behave as a negative mechanical damping as viewed by the rotor system. If this negative damping is greater than the electrical system positive damping then torque oscillation will build until the shaft is lighty stressed. This phenomenon is known as the torsional resonance interaction.

Mathematical Formalization. The linear model of the synchronous machine can be obtained as follows (El-Abiad 1983; Anderson *et al* 1990; El-Serafi and Badr 1976).

$$\begin{bmatrix}\Delta\psi_d\\ \Delta\psi_q\\ \Delta\psi_{fd}\\ \Delta\psi_{kd}\\ \Delta\psi_{kq}\end{bmatrix} = \begin{bmatrix}-x_d & 0 & x_{ad} & x_{ad} & 0\\ 0 & -x_q & 0 & 0 & x_{aq}\\ -x_{ad} & 0 & x_{ffd} & x_{fkd} & 0\\ -x_{ad} & 0 & x_{fkd} & x_{kkd} & 0\\ 0 & -x_{aq} & 0 & 0 & x_{kkq}\end{bmatrix}\begin{bmatrix}\Delta i_d\\ \Delta i_q\\ \Delta i_{fd}\\ \Delta i_{kd}\\ \Delta i_{kq}\end{bmatrix} \quad \text{..............(1)}$$

$$\begin{bmatrix}\Delta e_d\\ \Delta e_q\\ \Delta e_{fd}\\ 0\\ 0\end{bmatrix} = [r]\begin{bmatrix}\Delta i_d\\ \Delta i_q\\ \Delta i_{fd}\\ \Delta i_{kd}\\ \Delta i_{kq}\end{bmatrix} + \frac{1}{\omega_o}\frac{d}{dt}\begin{bmatrix}\Delta\psi_d\\ \Delta\psi_q\\ \Delta\psi_{fd}\\ \Delta\psi_{kd}\\ \Delta\psi_{kq}\end{bmatrix} + \omega_o\begin{bmatrix}-\Delta\psi_q\\ \Delta\psi_d\\ 0\\ 0\\ 0\end{bmatrix} + \Delta\omega\begin{bmatrix}-\psi_{qo}\\ \psi_{do}\\ 0\\ 0\\ 0\end{bmatrix} \quad \text{.......(2)}$$

where ψ_{qo} and ψ_{do} are the steady state values.

The linearized torsional equations of the machine shaft assembly for Fig 1 can be written in matrix form as follows:

$$\begin{bmatrix}\Delta\dot{\omega}_g\\ \Delta\dot{\delta}_g\\ \Delta\dot{\omega}_t\\ \Delta\dot{\delta}_t\end{bmatrix} = \begin{bmatrix}\frac{-(D_s+D_{tg})}{\theta g} & \frac{-k_{tg}}{\theta g} & \frac{D_{tg}}{\theta_g} & \frac{K_{tg}}{\theta_g}\\ \frac{D_{tg}}{\theta_t} & \frac{K_{tg}}{\theta_t} & \frac{-(D_{tg}+D_{tg})}{1} & \frac{-K_{tg}}{\theta_t}\\ 0 & 0 & 1 & 0\end{bmatrix} x \begin{bmatrix}\Delta\omega_g\\ \Delta\delta_g\\ \Delta\omega_t\\ \Delta\delta_t\end{bmatrix} - \begin{bmatrix}\Delta T_E\\ 0\\ 0\\ 0\end{bmatrix} \quad \text{......(3)}$$

The term Δ_{TE} can be written as:

$$\Delta T_E = \Delta\psi_d I_{go} + \Delta i\, g\psi_{do} - \Delta\psi_g I_{do} - \Delta i\, g\psi_{go} \qquad (4)$$

where I_{do}, I_{go} are steady state currents.
And the network linearized model for Fig. (1) is modelled by

$$\begin{bmatrix}\Delta e_D\\ \Delta e_Q\end{bmatrix} = \begin{bmatrix}r_E & -X_E\\ X_E & r_E\end{bmatrix}\begin{bmatrix}\Delta i_D\\ \Delta i_Q\end{bmatrix} + \frac{1}{\omega_o}\begin{bmatrix}X_E & 0\\ 0 & X_E\end{bmatrix}\frac{d}{dt}\begin{bmatrix}\Delta i_D\\ \delta i_Q\end{bmatrix} \quad \text{.......(5)}$$

and

$$\begin{bmatrix}\Delta e_d\\ \Delta e_q\end{bmatrix} = \begin{bmatrix}\cos\sigma_o & \sin\sigma_o\\ -\sin\sigma_o & \cos\sigma_o\end{bmatrix}\begin{bmatrix}\Delta e_D\\ \Delta e_Q\end{bmatrix} + \Delta\sigma\begin{bmatrix}-\sin\sigma_o & \cos\sigma_o\\ -\cos\sigma_o & -\sin\sigma_o\end{bmatrix}\begin{bmatrix}\Delta e_{Do}\\ \delta e_{Qo}\end{bmatrix} \quad \text{.......(6)}$$

Also $\Delta\sigma$ equal $\Delta\delta$.
The network equations of the system in Fig (1) are written as:

$$\frac{1}{\omega_o}\frac{d}{dt}\begin{bmatrix}\Delta e_{CD}\\ \Delta e_{CQ}\end{bmatrix} = \begin{bmatrix}X_C & 0\\ 0 & X_C\end{bmatrix}\begin{bmatrix}\Delta i_D\\ \Delta i_Q\end{bmatrix} + \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}\Delta e_{CD}\\ \Delta e_{CQ}\end{bmatrix} \quad \text{.......(7)}$$

where e_{CD} and e_{CQ} are voltage of the series capacitor in D-Q axes. So the DQ linearized model for synchronous machine in Fig 1 can be written in a matrix from as in equation 8

$$\begin{bmatrix}-X_d-X_E & 0 & X_{ad} & X_{ad} & 0 & 0 & 0\\ 0 & -X_q-X_E & 0 & 0 & X_{aq} & 0 & .0\\ -X_{ad} & 0 & X_{ffd} & X_{fkd} & 0 & 0 & 0\\ -X_{ad} & 0 & X_{fkd} & X_{kkd} & 0 & 0 & 0\\ 0 & -X_{aq} & 0 & 0 & X_{kkq} & 0 & 0\\ & & & & & \frac{2H}{\omega_o} & \\ & & & & & & 1\end{bmatrix}\frac{d}{dt}\begin{bmatrix}\Delta i_D\\ \Delta i_Q\\ \Delta i_{fd}\\ \Delta i_{kd}\\ \Delta i_{kq}\\ \Delta\omega\\ \Delta\delta\end{bmatrix} = \quad \text{.......(8)}$$

$$\begin{bmatrix}r_a+r_E & -X_q-X_E & & & X_{aq} & A & -B\\ X_d+X_E & r_a+r_E & -X_{ad} & -X_{ad} & & B & A\\ & & -r_{fd} & & X_{ad}xI_{qo} & & \\ & & & -r_{kd} & & X_{ad}XX_{qo} & \\ & & & & -r_{kq} & -X_{kq}xI_{do} & \\ A & B & -X_{ad}I_{qo} & -X_{ad}xI_{qo} & X_{aq}I_{do} & & P_s\\ & & & & & 1 & \end{bmatrix}\begin{bmatrix}\Delta i_D\\ \Delta i_Q\\ \Delta i_{fd}\\ \Delta i_{kd}\\ \Delta i_{kq}\\ \Delta\omega\\ \Delta\delta\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \Delta e_{fd}\\ 0\\ 0\\ \Delta T_M\\ 0\end{bmatrix}$$

So the overall linearized mathematical model describing the series compensated power system shown in Fig 1 can be obtained from equations 5, 6, 7 and 8 and have to be in this form where x is state variable vector that has the current, speed, angle and the series capacitor voltage. [F] and [D] are constant matrices remembering that $\Delta Tm = 0$ for dynamic stability.

$$\frac{1}{\omega_O}[D]\frac{d}{dt}X = [F]\, X + v \qquad (9)$$

Also Δe_{fD} in equation 8 is incorporated in overall model, Eq. (9) will have the form of Eq. (10).

The system is said to be dynamically stable if none of the real parts of the complex eigenvalues of matrix [A] is positive. where $[A] = \omega o\,[D]^{-1}\,[F]$

$$\frac{d}{dt} X = [A]\, X \qquad (10)$$

The numerical method used to examine the dynamic stability of this system is the eigenvalue calculation of the transfer matrix [A]. The system is said to be dynamically stable if none of the real parts of the complex eigenvalues of matrix [A] is positive.

Computer Simulation Results. A digital computer has been developed (El-Sadany 1990) to form the overall characteristic matrix of the system at each operating point and then to check its stability by applying the eigenvalue technique. The results obtained are as of the curve representing the dynamic stability that can be drawn in the plane of any two arbitrary parameters. Impact of subsynchronous resonance on dynamics' stability can be obtained by varying the degree of compensating Xc and the external line resistance R_E. This is justifiable since both parameters play an important role in determining the subsynchronous resonance unstable zone. Figure 2 show the stability boundaries in R_E-X_E plane.

Zone A shows the instability associated with the phenomena of electrical self excitation. Negative damping here is attributed to the presence of a negative total resistance in the armature circuit. Instability in zone B is contributed by slow electromechanical oscillations hunting mode governed mainly by the resultant inertia and synchronizing torque coefficient. It is worth to mention here that the instability in Zone C is due to the subsynchronous resonance between the electrical oscillation of the line and torsional oscillation of the turbine shaft assembly.

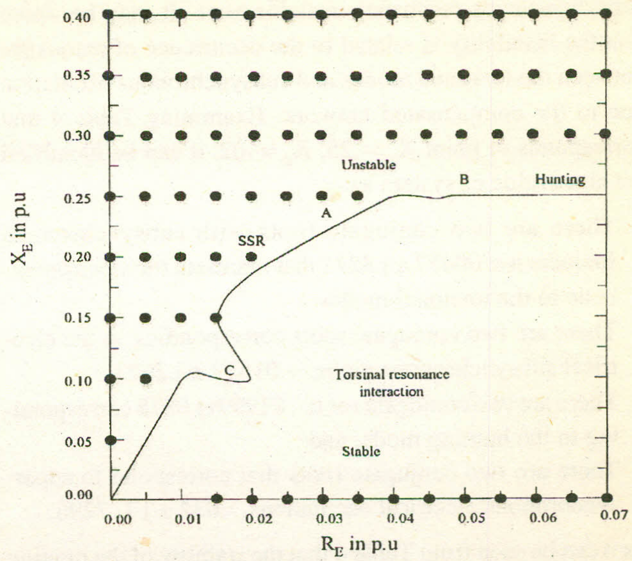

Fig 2. Stability boundary in the X_C - R_E plane.

A typical output from several runs has been conducted using eigenvalue program to determine the system eigenvalues under specific conditions. The system eigenvalues are shown in Table 4. The electrical system eigenvalues appear as the 60 Hz complement.

The eigenvalues associated with the mechanical system can be easily identified since their imaginary parts (frequencies) are expected is advance. These frequencies do not change appreciably with various systems operating conditions but the real parts of the eigenvalues will often change.

Table 4 shows the eigenvalue of mentioned system for points of compensation $X_c = 0.00001$, and $R_E = O$, $X_c = 0.25$, $R_E = 0.02$, and $X_c = 0.35$, $R_E = 0.05$ p.u. As can be seen system is unstable at $X_c = 0.25$, 0.35 while it is stable at $X_c = 0.0001$ p.u.

The instability points here correspond to the points of

Table 4

Chosen eigenvalue of the system under different compensation in p.u.

Mode identification	$R_E = 0$ Xc =.00001	$R_E = 0.15$ Xc = 0.15	R_E =.02 Xc =.25	R_E = .05 Xc = .35
Supersynchronous electrical mode	-.00017 ±j.1.00	-0.0258 ± j 1.564	-.032 ± 1.7293	-.0706 ± j1.8617
Subsynchronous electrical mode	.012 ±j.997	+0.00019 ±j0.43369	+.01622 ±j.2637	+.03685 ±j.1359
Torsional mode	-.00081 ±j.832	-0.0048 ±j0.8826	-.00457 ±j.8271	-.00452 ±j.8277
Hunting mode	-.00504 ±j.0788	-0.01 ±j0.0857	-.01999 ±j.0878	-.02745 ±j.06507

subsynchronous resonance unstable zone. It may be stated that the instability is related to the occurrence of resonance between the torsional modes and subsynchronous oscillation due to the compensated network. Examining Table 4 that corresponds to point $X_c = .25$, $R_E = .02$, it can be identified the eigenvalue of system as:-

- There are two conjugate roots with subsynchronous frequencies. 00457 ± j.8271 that represent the electromagnetic to the torsional modes.
- There are two conjugate roots corresponding to the electrical subsynchronous mode, +.01622 ± j.2673.
- There are two conjugate roots-. 01999±j.0878 corresponding to the hunting mode, and
- There are two conjugate roots that correspond to supersynchronous electrical oscillations, -.032 ± j 1. 7296.

As it can be seen from Table 4 that the stability of the hunting mode is improving with increasing the level of compensation. The instability associated with the column 2, 3 and 4 is due to the subsynchronous mode.

Conclusion

This paper presents an experimental study of subsynchronous resonance behaviour of a small rating motor-generator set connected to infinite bus through series capacitor bank. The result is complemented by theoretical verifications using rigorous mathematical modelling and computer simulation. From these studies it can be concluded that:

1) Experiments show that due to subsynchronous resonance the machine set undergoes severe torsional vibrations that caused the machine to go out of synchronism.
2) The above experimental fact has been confirmed by computer simulation of the dynamic stability problem aimed at determining subsynchronous resonance unstable zones. A good correlation was obtained between measured and calculated torsional natural frequencies.
3) It is found necessary to analyze the torsional resonance whenever series capacitors are applied in the vicinity of a turbine-driven synchronous generator.

List of Symbols

i	Generator armature current
i_{fd}	Field current
V	Infinite bus bar voltage
V_t	Generator terminal voltage
V_{fd}	Field voltage
r_a	Armature resistance
R_e	External line resistance
r_{fd}	Field winding resistance
r_{kd}, r_{kq}	d-and q-axis damper winding resistance, respectively
X_{ad}, X_{aq}	d-and q-axis magnetizing reactance, respectively
X_d, X_q	d-and q-axis synchronous reactance, respectively
X_d, X_q	d-and q-axis transient reactance, respectively
X_d, X_q	d-and q-axis subtransient reactance, respectively
ω_t, δ_t	Turbine speed and angle, respectively
ω_o	Synchronous speed
ω_n, ω_m	Subsynchronous and mechanical mode speeds, respectively
ψ_d, ψ_q	d-and q-axis armature flux linkage, respectively
ψ_{fd}	Field winding flux linkage
ψ_{kd}, ψ_{kq}	d-and q-axis damper winding flux linkage, respectively
p^{θ}	Speed of the machine (elect. rad/sec)
X_{ffd}	Field winding self reactance
X_c	External line reactance
X_{kd}, X_{kq}	D-and q-axis damper winding self reactance respectively
D_{ee}, D_{gg}	Damping coefficients of the corresponding inertia
D_{ge}	Mutual damping between the exciter and generator
D_{GA}	Mutual damping between turbine-A and genecator
D_{AB}	Mutual damping between turbine-A and turbine-B
K_{GE}, K_{AB}	Stiffness of the connected shaft
θ_g, θ_e	Inertial constants
K	Electromagnetic torque constant
K_{eq}	Lumped masses respectively
K_E	Exciter gain
T_E	Exciter time constant
T_{do}	d-axis open circuit transient time constant
T_s	Synchronizing torque
T_D	Damping torque

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