Pak. j. sci. ind. res., vol. 36, no. 10, October 1993

E2/M1 MULTIPOLE ADMIXTURES OF GAMMA TRANSITIONS IN EVEN-EVEN NUCLEI

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(Received July 8, 1992; revised May 26, 1993)

Measurement of E2/M1 multipole mixing ratios of transitions in even-even nuclei have long provided important tests of nuclear models. The available experimental data on E2/M1 multipole mixing ratios of gamma transitions de-exciting levels of beta and gamma vibrational bands to the ground state rotational band in even-even nuclei with 150 A 200 have been reviewed. The results have been compared to the predictions of the dynamic deformation model and the interacting boson model.

Key words: Mixing ratios, Gamma-Gamma (θ), Deduced δ.

Introduction

When the nuclear spin-parity selection rules allow more than one multipole radiation in a gamma transition, only the lowest two multipole orders have been observed to compete. In principle any multipole order between the sum and difference of the initial and final nuclear spins is allowed but the strong dependence of the transition probabilities on the angular momentum carried off by the gamma radiation restricts the observable orders to the lowest ones. Generally, the lowest of the two multipoles dominates. However, when the selection rules allow, E2 radiation often dominates the M1 radiation. This is because the nuclear structure effects override the angular momentum dependence of the transition probabilities. Thus experimental determinations of the admixtures of E2/M1 radiations in nuclear transitions, particularly in even-even nuclei, have provided for over three decades many significant tests of nuclear models. Because of the importance of these data, there have been periodic surveys of E2/M1 mixing ratios and theoretical studies [1-7]. The most recent comparison of theory and experiment for E2/M1 admixed transitions from all classes of nuclei have been made by Lange *et al.* [7] who surveyed the experimental data upto 1980.

The present paper presents a survey of the experimental data upto 1990 for even-even nuclei with 150 A 200. The experimental results are then compared with the predictions of the Dynamic Deformation Model (DDM) of Kumar [8] and the Interacting Boson Model (IBM) of Iachello and Arima [9]. These models have shown remarkable success in predicting the multipole mixing ratios.

The E2/M1 mixing ratio δ(E2/M1). There are several definitions of the multipole mixing ratio δ in the literature [10-13]. All of these definitions differ in the sign conventions. The measurement of gamma-ray angular distributions is sensitive to the interference effects between the E2 and M1 amplitudes, and thus depends on the relative sign of the E2 and M1 matrix elements. A number of different conventions have been used in the literature to relate this phase to the observed angular distributions and hence several definitions of the mixing ratio. In the present work, the phase convention proposed by Krane and Steffen [13] is used. According to this convention the mixing ratio δ (E2/M1) is defined (in terms of matrix elements of the Bohr-Mottelson multipole operators [14]) as:

$$\delta(E2/M1) = 0.835 E_\gamma (Mev) \frac{\langle I_f \,||\, M(E2) \,||\, I_i \rangle \, eb}{\langle I_f \,||\, M(M1) \,||\, I_i \rangle \, nm} \quad ....(1)$$

A comprehensive discussion of the properties of the electro-magnetic transition operators and their matrix elements is given in the earlier literature [14].

Experimental

The E2/M1 mixing ratios are deduced experimentally from the analysis of the angular distribution of the gamma rays. The angular distribution probability depends on the way the axis of the alignment is defined. The most common method consists of the measurement of the angular correlation of gamma rays emitted in a cascade.

$$I_1 \xrightarrow{\gamma_1} I_2 \xrightarrow{\gamma_2} I_3 \quad .... (2)$$

The corresponding relation for the angular distribution probability (correlation function) [15,16] is

$$W(\theta) = \sum_{k=even} A_k (\gamma_1) A_k (\gamma_2) P_k (Cos\theta) \quad .... (3)$$

or in the normalized form ($A_0 = 1$)

$$W(\theta) = 1 + \sum_{k=2,4,...} A_k (\gamma_1\gamma_2) P_k (Cos\theta) \quad .... (4)$$

where $A_k (\gamma_1\gamma_2) = A_k (\gamma_1) A_k(\gamma_2)$ and $A_0 (\gamma_1) = A_0 (\gamma_2) = 1$

In practice only coefficients through A_4 are necessary for dipole (L=1) and quadrupole (L=2) radiations. The correlation coefficients $A_k(\gamma_1)$ and $A_k(\gamma_2)$ are defined as

$$A_k(\gamma_1) = \frac{[F_k(11I_1I_2) - 2\delta(\gamma_1)F_k(12\,I_1I_2) + \delta^2(\gamma_1)F_k(22\,I_1I_2)]}{1 + \delta^2(\gamma_1)} \qquad (5)$$

$$A_k(\gamma_2) = \frac{[F_k(11I_3I_2) + 2\delta(\gamma_2)F_k(12\,I_3I_2) + \delta^2(\gamma_2)F_k(22\,I_3I_2)]}{1 + \delta^2(\gamma_2)} \qquad (6)$$

The geometrical factors F_k have been defined and tabulated by Frauenfelder and Steffen [15].

The correlation coefficients A_k contain all the physical information and they depend on the spins of the nuclear states and on the type (electric or magnetic) and multipolarity of the radiations involved. These coefficients are determined experimentally from the analysis of the measured coincidence counting data as a function of the angle θ between the directions of emission of the two radiations. A comparison of the experimentally determined correlation coefficients with theoretical values (Eqs. 5, 6) helps determine the multipole mixing ratios of the transitions and delimits the spins of the nuclear levels involved.

A careful review of the available data obtained from angular correlation measurements of gamma rays upto 1990 was made. Experimentally the mixing ratio is determined from the A_2 and A_4 coefficients. Generally both the coefficients and the deduced δ values are reported in the literature. But in some cases only the coefficients A_2 and A_4 are reported. In such cases the δ values were then determined by the standard method of comparison with theoretical values.

The δ values reported in the literature were obtained through gamma rays angular correlation measurements with NaI (T1)–NaI(T1), NaI(T1)–Ge(Li) and Ge(Li)-Ge(Li) detector systems. Only the values obtained with the measurements made with NaI(T1)-Ge(Li) and Ge(Li)–Ge(Li) detector systems were considered in the present work.

A summary of the best results obtained from an analysis of the available angular correlation literature [1-7, 17-22] in terms of the present phase convention is given in the last column in Table 1. Even parity transitions depopulating states of the β and γ bands with 1 4 have been analysed for even-even nuclei with 150 A 200.

The systematic behaviour of the phase of the mixing ratio is apparent from an inspection of the table. With minor exceptions, transitions from the γ band have negative phase, while a majority of the transitions from the β band show the opposite phase.

Result and Discussion

Model predictions of the mixing ratios. Although a large number of models have been developed and applied to the

TABLE 1. COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF E2/M1 MIXING RATIOS.

		δ(Theory)			
Nucleus	Transition	Eγ(Mev)	IBM	DDM	δ(Expt)
Sm^{150}	2_β—2	0.712	6.63	-4	-4.8± 0.5
	4_β—4	0.676	3.74	-10	-1.3±0.3
	2_γ—2	0.860	-5.30	58	3.4±0.7
	3_γ—2	1.171	-7.10	-221	3.6±1.3
	3_γ—4	0.731	-3.44	127	13(∞,—7)
	4_γ—4	0.869	-3.20	33	|δ| 0.7
Sm^{152}	2_β—2	0.689	6.42	11	19± 4.50
	4_β—4	0.657	3.64	4	2.1±0.30
	2_γ—2	0.695	-5.94	-24	-9.6±0.30
	3_γ—2	1.113	-6.75	-27	-8.7±0.60
	3_γ—4	0.869	-4.10	-16	-6.5±0.30
	4_γ—4	1.005	-3.68	-10	-3.1±0.30
Gd^{152}	2_β—2	0.586	5.46	-7	-3.0±0.30
	2_γ—2	0.765	-4.71	28	4.3±0.60
Gd^{154}	2_β—2	0.692	6.45	1.04	8.3±1.20
	4_β—4	0.676	3.74	-0.56	3.0±1.30
	2_γ—2	0.873	-5.40	-14.6	-9.7±0.5
	3_γ—2	1.005	-6.10	-10.3	-7.6±0.40
	3_γ—4	0.757	-3.56	-2	-5.6±0.20
	4_γ—4	0.89	-3.26	-12	-4.0±0.40
Gd^{156}	2_β—2	1.040	9.70	21	-14(∞,-7)
	2_γ—2	1.065	-6.60	-41	-17.5±1.5
	3_γ—2	1.159	-7.00	-57	-8.60±3.1
	3_γ—4	0.960	-4.50	-37	-12(14,-3)
	4_γ—4	1.067	-3.90	14	-4.0±1.20
Dy^{160}	2_γ—2	0.879	-5.40	–	-14.8±11
	3_γ—2	0.692	-4.20	–	-12±2.5
	3_γ—4	0.765	-3.60	–	-5.3±2.8
Dy^{162}	2_γ—2	0.08	-4.97	–	|δ| 20
Dy^{164}	2_γ—2	0.689	-4.24	–	-7(∞,-4)
Er^{166}	2_γ—2	0.705	-4.34	-9.42	-16(13,-5)
	3_γ—2	0.779	-4.73	-41	-19(190, -9)
	3_γ—4	0.593	-2.80	-29.2	-9 (∞,-5)
	4_γ—4	0.691	-2.53	3.63	-16(27,-4)
Er^{168}	2_γ—2	0.742	-4.56	–	|δ| 87
Er^{170}	2_γ—2	0.853	-5.25	–	-55(∞,-34)
	2_β—2	0.881	8.20	–	1.7±0.8
Yb^{172}	2_γ—2	1.387	-8.54	–	-5.1±1.4
	3_γ—4	0.912	-4.28	–	-2.36±0.15
	3_β—4	1.289	9.17	–	2.8±0.8
	2_β—2	0.809	7.54	–	-11 (∞,-7)
Hf^{174}	4_β—4	0.765	4.23	–	-2.5±0.8
Hf^{176}	2_β—2	1.138	10.78	–	|δ| 4

(Table 1, cont'd...)

(Table 1, continue...)

Hf^{178}	2_β—2	1.403	13.07	–	-0.75±15
	2_γ—2	1.081	-6.65	–	|δ| 11
Hf^{180}	2_γ—2	1.107	-6.80	–	9.6(22,-5.8)
W^{182}	2_β—2	1.157	-10.78	17	-8.7±3.5
	2_γ—2	1.122	-6.90	-5	16.7±3.4
W^{184}	2_β—2	1.275	11.88	-2	28 (∞,-17)
	2_γ—2	0.793	-4.88	45	-16.7±0.5
W^{186}	2_β—2	1.164	10.84	-2	13 (70, -6)
	2_γ—2	0.615	-3.78	-218	-11±3.5
	2_γ—2	0.630	-3.88	-15	-50 (∞, -30)
	2_γ—2	0.773	-4.70	-14	-13 (9, -6)
Os^{188}	2_γ—2	0.478	-2.94	-10	-23 (9, -5)
	3_γ—2	0.635	-3.85	-11	-6.9±1.8
Os^{190}	2_γ—2	0.371	-2.28	-8	-9.2±0.7
	3_γ—2	0.569	-3.45	-10	-9.4±1.4
Os^{192}	2_γ—2	0.283	-1.74	15	-4.2±0.4
	3_γ—2	0.484	-2.94	-2	-9.2±0.8
Pt^{192}	2_γ—2	0.296	-1.82	15	8.84±0.26
	3_γ—2	0.604	-3.66	-2	-1.82±0.12
Pt^{194}	2_γ—2	0.294	-1.81	20	19±4
Pt^{196}	2_γ—2	0.335	-2.05	-101	4.8±0.2

study of energy levels, B(E2) values, and static moments, only a few have been employed for the prediction of the mixing ratios. The DDM [8] and IBM [9] have achieved remarkable success in predicting the multipole mixing ratios. We consider here predictions of these two models for comparison with experimental results. A brief description of these models is given below.

The dynamic deformation model (DDM). The dynamic deformation model has been developed by Kumar [8], starting with the pairing-plus-quadrupole (PPQ) model of Kumar and Baranger [23], in an attempt to achieve a unified theory of light, medium and heavy nuclei irrespective of their equilibrium shape. It is a microscopic version of the collective model of Bohr and Mottelson [14] and combines the dynamic treatment of nuclear deformations with a better microscopic theory, where the quadrupole-quadrupole interaction part of the PPQ model is replaced by the Nilsson-Strutinsky method [24].

The multipole operators $\mathcal{M}$(E2) and $\mathcal{M}$(M1) of Eq. [11] are defined microscopically as

$$\mathcal{M}(E2, \mu) = \sum_{i=1}^{z} r_i^2 \, Y_{2\mu}(\theta_1, \phi_i) \qquad (7)$$

$$\mathcal{M}(M1,\mu) = \sum_{i=1}^{z} l_{i\mu} + \sum_{\tau=n,p} g_{s\tau} \sum_{i=1}^{A} S_{i\mu} \qquad (8)$$

It has been shown that this model is capable of producing the low energy properties of parctically all even-even nuclei without any adjustment of parameters from nucleus to nucleus [8].

The interacting boson model. The interacting boson model (IBM) has been developed by Iachello and Arima [9]. It was introduced in an attempt to describe in a unified way collective properties of nuclei. This model is rooted in the spherical Shell Model developed by Jenson and Mayer [25], which is the fundamental model for describing properties of nuclei, but in addition has properties similar and in many cases identical, to the collective model of Bohr and Mottelson [14] based on the concept of shape variable.

In this model, the low energy states of even-even nuclei are described in terms of interactions between s(L=0) and d(L=2) bosons. The corresponding hamiltonian is diagonalized in this boson space by employing group theory methods.

Two of the versions of the IBM are called IBM-1 and IBM-2. The IBM-2 distinguishes between neutron and proton bosons and includes four types of bosons: one set of (s,d) for neutrons and a second set for protons. We consider here the IBM-1.

The multipole operators $\mathcal{M}$(E2) and $\mathcal{M}$(M1) (Eq. [11]), denoted by T(E2) and T(M1), in the IBM are given by

$$T(E2) = \alpha_2 \, [ds + sd]_2 + \beta_2 \, [dd]_2 \quad \ldots\ldots \quad (9)$$

$$T(M1) = [g_b + AN]L + B_1 \, [(ds + sd)\, L]_1 + B_2 \, [\, (dd)_2 \, L]_1 + C n_d L \quad \ldots\ldots \quad (10)$$

The M1 operator must be taken to second order, since in first order it is proportional to the angular momentum L and thus incapable of yielding M1 transitions.

The values of the mixing ratios δ (E2/M1) calculated with DDM and IBM-1, for even-even nuclei in the region 150 <A< 200 are given in Table 1. The DDM results for δ were taken from [7,8,21,22] while the IBM-1 results were calculated using the computer program PHINT [26].

Comparison of the experimental and theoretical results. The experimental values of the mixing ratios as discussed in experimental and the theoretical values obtained from DDM and IBM as discussed in Results and discussion are given in Table 1 for comparison.

The general features that the β→g and γ→g transitions are largely E2 is shown by both the experimental and theoretical results.

The DDM calculated values of δ for γ→g transitions are large (M1 components are too small) but the signs and general trends are given correctly. The IBM gives better agreement with the magnitudes but not the signs of the mixing ratios.

The mixing ratio δ (E2/M1)provides an important probe into the nuclear structure and is in fact regarded as a measure of collectivity of the nucleus. The experimental measurements of the E2/M1 mixing ratios have been a productive tool for understanding the structure of the nucleus and have provided many significant tests of nuclear models. The results of the Dynamic Deformation Model and the Interacting Boson Model are encouraging. The former is a microscopic model while the latter is a phenomenalogical model. The DDM and IBM provide a hope for a better understanding, if not complete theory, of the nucleus in future. The discrepency in the experimental and theoretical values of the mixing ratios could probably be removed by further refinement in the theory.

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