

Short Communication

Pak. j. sci. ind. res., vol. 35, no. 9, September 1992

**The Heat Capacity,  $C_v$ , for  $Co^{2+}/MgO$  Single Crystal**

M.D. HOSSAIN AND M.R. ISLAM

*Applied Physics and Electronics Department  
University of Rajshahi, Rajshahi - 6205, Bangladesh*

(Received December 15, 1991; revised July 6, 1992)

The heat capacity  $C_v$  for  $Co^{2+}/MgO$  single crystal as a function of temperature has been evaluated from the elastic constants data. These values agree well with those obtained from the calorimeter measurement. From the heat capacity data the entropy at 300K is estimated to be  $20.86 J \cdot mol^{-1} \cdot K^{-1}$ .

Advanced systems for energy conversion, such as the magnetohydrodynamic generator, are generally designed to be operated at high temperatures. A serious obstacle to long-term performance of these systems is the degradation of the material used for electrical insulation [1]. Refractory insulators must be available which can withstand high-temperature operation without material degradation, such as electrothermal breakdown. It is therefore, important to recognize the parameters associated with the degradation - especially those which can improve the long-term characteristics. The impurity content is a parameter which has been known to shorten the characteristic times for breakdown [2- 3].  $MgO$  is a refractory material of high chemical stability. It has been reported by Tsang and Chen [4] that the presence of Cu, Co, or H in  $MgO$  crystal suppresses the electrothermal breakdown and prolongs the lifetime of  $MgO$  crystal. In the case of crystals doped with 2000 ppm Co, the characteristic time for breakdown is 10 times longer than that of undoped (pure) crystals [4]. Heat capacity at constant pressure of single crystal  $MgO$  (pure) has measured from 300°k to 1200°k [5] and down to 20°k [6] by using calorimeter method. To observe the presence of impurities on the heat capacity, it has been decided to estimate the specific heat capacity  $C_v$  (at constant volume) for cobalt doped  $MgO$  single crystal, from the reported elastic constant data given by M.D. Hossain [7] which are measured from the Ultrasonic pluse-Echo Technique. Apart from this, the purpose of the present work is to evaluate the internal energy as well as entropy of  $MgO$  single crystals. No previous results on  $Co^{2+}/MgO$  in this regard have seemingly been published.

*Specific heat capacity  $C_v$  (working formula).* In 1912, Debye [8] realized that it is possible to propagate elastic waves through solids covering a wavelength region extending from low frequencies (sound waves) upto short waves (infrared

absorption). According to him, the specific heat capacity,  $C_v$ , is given by the following expression:

$$C_v = 9NK_B Z (T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x - 1)} dx \dots\dots\dots(1)$$

where  $\theta_D$ , is known as a Debye characteristic temperature, N is the Avogadro number,  $K_B$  is the Boltzmann constant, T is the absolute temperature and Z is the number of atoms in one molecule. The temperature  $\theta_D$  is an important parameter of a solid. It is found in equations describing properties which arise from atomic vibrations and in theories involving phonons.

Recently, the author has reported [7] the experimental results of the elastic constant of  $Co^{2+}/MgO$  (9900 ppm) by ultrasonic pulse echo-overlap technique at room temperature. The values of these constants are as  $C_{11} = 28.50 \times 10^{11}$  dynes/cm<sup>2</sup>,  $C_{12} = 8.61 \times 10^{11}$  dynes/cm<sup>2</sup> and  $C_{44} = 15.21 \times 10^{11}$  dynes/cm<sup>2</sup>. One of the standard methods of calculating the Debye temperature  $\theta_D$  is from the elastic constant data, since  $\theta_D$  is proportional to the sound velocity (averaged)  $\bar{V}_m$ . The relation between  $\theta_D$  and  $\bar{V}_m$  is shown in section III, and  $\bar{V}_m$  with the elastic constants is shown in section IV.

*Relation between debye temperature  $\theta_D$  and averaged sound velocity  $\bar{V}_m$ .* It has been shown by Huntington [9] that the Debye temperature,  $\theta_D$ , is proportional to the mean sound velocity,  $\bar{V}_m$  of a solid in the form

$$\theta_D = \frac{h}{K_B} \left( \frac{3ZN\rho}{4\pi M} \right)^{1/3} \bar{V}_m \dots\dots\dots(2)$$

where h is Planck's constant, N is Avogadro number, M is the molecular weight of the solid, and  $\rho$  is the density. {For  $Co^{2+}/MgO$ ,  $Z = 2$ ,  $\rho = 3.61$  gm per cm<sup>3</sup> [7]}.

The use of Equation -2 is often hindered by the problem of computing  $\bar{V}_m$ . Since  $\theta_D$  is scalar, it follows that  $\bar{V}_m$  must also be a scalar, and herein lies the difficulty of this equation. The stress is a tensor quantity and for each direction in a crystal, there are three velocities each of which is a complicated function of the stress components. The expression for  $\bar{V}_m$  is given by Huntington [9] is

$$\bar{V}_m = \left[ \frac{1}{3} \sum_{i=1}^3 \int_0^{4\pi} \frac{1}{V_i^3} \frac{d\Omega}{4\pi} \right]^{1/3} \dots\dots\dots(3)$$

Equation -3 can be solved for a solid only by a rigorous computation based on the precise knowledge of the elastic

constants of the crystal. Anderson [10], however, showed that Equation -3 can be substituted by the following simple equation to determine the mean sound velocity for all solids.

$$\bar{V}_m \left[ \frac{1}{3} \left( \frac{1}{V_s^3} + \frac{1}{V_l^3} \right) \right]^{-1/3} \dots\dots\dots(4)$$

where  $V_s$  and  $V_l$  are the mean shear and longitudinal sound velocities respectively. According to Gilvarry [11] the parameters  $V_s$  and  $V_l$  can be defined by the following formula.

$$\bar{V}_s = \left[ \frac{G_H}{\rho} \right]^{1/2} \dots\dots\dots(5)$$

$$\bar{V}_l = \left[ \frac{K_H + \frac{3}{4}G_H}{\rho} \right]^{1/2} \dots\dots\dots(6)$$

where the symbols  $G_H$  and  $K_H$  represent bulk modulus and shear modulus for the cubic single crystal. The temperature dependence heat capacity  $C_v$  is then calculated by using equation 1 and 2.

**Specimen history:** The doped single crystal of  $Co^{2+}/MgO$  was obtained from W&C Spicer Ltd. (U.K.) having been grown by electrofusion using pure powdered cobalt oxide and pure powder magnesia as starting materials. The cobalt concentration in the specimen was 9900 ppm, this had been determined by X-ray fluorescent analysis (Johnson and Mathey Ltd.). The crystalline quality was good and neither optical examination nor X-ray back reflection photograph, used to orient the specimens, revealed any evidence of macroscopic cracking, flaws, strain or mosaic formation. The colour of the crystal was deep pink. ESR measurements on this specimen [12] showed only line due to  $Co^{2+}$  which suggests that the crystal has got no other impurities. This is due to the fact that, since  $Co^{2+}$  ion is doubly charged, a direct substitution for  $Mg^{2+}$  is possible in the  $MgO$  lattice without the need for charge compensating vacancies.

**Ultrasonic pulse-echo overlap technique.** The elastic constants of  $Co^{2+}/MgO$  were determined from the ultrasonic velocity measurements by pulse echo-overlap method. Ultrasonic wave pulses were generated and received with X-cut (for longitudinal waves) and Y-cut (for shear waves) quartz transducers of 6 mm diameter with a resonance frequency of 15 MHz as described by M.D. Hossain [7]. In this system the R.F. Unit, which delivers pulses to the sample and the resultant unrectified echo train is displayed on the oscilloscope. The RF generator is rigged by the frequency synthesiser output divided by one hundred or one thousand, so that any given echo train has completely decayed before the next RF pulse. The technique now is to overlap two of the echoes by triggering from the frequency synthesiser. When the set frequency is an exact multiple of the inverse transit time, the echoes overlap

each other precisely. In this situation all the RF oscillations are in phase and the maxima of each echo coincident. The transit time for the RF pulse within the specimen is now the reciprocal of the trigger frequency. The distance between each successive peak on each train represents twice the transit time of the ultrasound pulse across the sample. Therefore once the sample length has been measured, the ultrasound velocity can be obtained as the distance travelled divided by the transit time.

By using the experimental values of elastic constants of  $Co^{2+}/MgO$  [7], the temperature dependence of heat capacity  $C_v$  in the temperature range up to 1200K, have been evaluated with the help of the equations given in the Section II and III. The calculated values of  $C_v$  for  $Co^{2+}/MgO$  are plotted in Fig. 1(a) by dotted line. The reported values of  $C_v$  for pure  $MgO$  single crystal shown in Fig. 1(b) by solid line, which are obtained from the measured values of  $C_p$  from the calorimeter method [5,6]. Comparison between the calculated results for the  $Co^{2+}/MgO$  single crystal and those reported from calorimeter method for  $MgO$  single crystal shows that both the results are very close to each other. This supports the validity of the present method of calculation of  $C_v$  from the elastic constant data. Though it has been shown in the literature [4] that the certain impurities do indeed enhance the electrical breakdown in  $MgO$  crystals, but on the other hand the present study shows that the heat capacity,  $C_v$  of  $MgO$  crystal is independent of divalent cobalt doped concentration.

The internal energy  $U$ , has been calculated by using the following equation:

$$U = 9 N K_B T (T/\theta_D)^3 \int_0^{\theta_D/T} \frac{x^3}{e^x - 1} dx \dots\dots\dots(7)$$

and the calculated values are shown in Fig. 1(c). These values are useful for the evaluation of free energy.

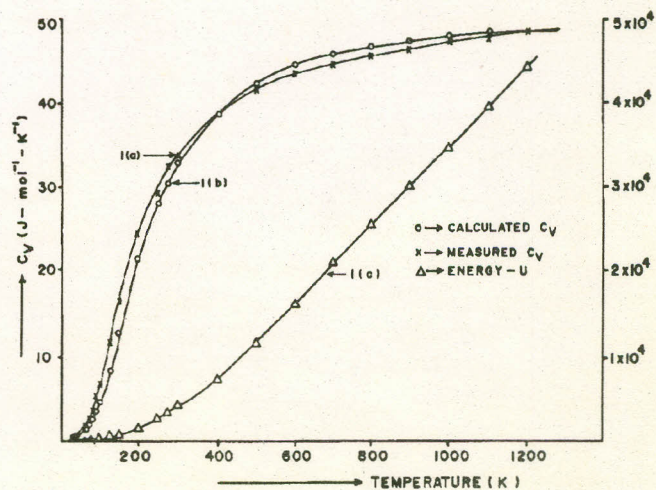


Fig. 1. Energy and heat capacity,  $C_v$  (for  $Co^{2+}/MgO$ ) with temperature.

The entropy  $S$ , at 300K is obtained from the relation,

$$S = \int_0^{300} \frac{C_V}{T} dT \dots\dots\dots(8)$$

$$= 20.86 \text{ J-mol}^{-1} \text{ -K}^{-1}$$

This is comparable to the literature value  $S=26.91 \text{ J.mol}^{-1} \text{ K}^{-1}$  of MgO at 300K [6] and thus again supports the validity of the present method of evaluation of  $C_v$ .

In conclusion, it can be said that the present study of heat capacity and internal energy as a function of temperature reveals additional information about the thermal properties of  $\text{Co}^{2+}/\text{MgO}$  single crystal and this knowledge may be helpful for understanding the various problems of conduction mechanism for materials of structure like MgO.

**Keywords:** Heatcapacity, Debye temperature, MgO single crystal.

#### References

1. Arthur Kantrowitz, ANL-77-21, 289 (1977).
2. E. Sonder, K.F. Kelton, J.C. Pigg and R.A. Weeks, J. Appl. Phys., **49**, 5971 (1978).
3. K.L. Tsang, Y. Chen and J.J.O. Dwyer, Phys. Rev., **B26**, 8909 (1982).
4. K.L. Tsang and Y. Chen, J. Appl. Phys., **54**, 4531 (1983).
5. A.C. Victor and T.B. Douglas, J. Res. Natl. Bur, Std., **67A**, 325 (1963).
6. T.H.K. Barron, W.T. Berg and J.A. Morison, Proc. Roy. Soc. London, **A244**, 70 (1959).
7. M.D. Hossain, Ind. J. Physics, **61A**, 143 (1987).
8. P. Debye, Ann. d. Physik, **39**, 789 (1912).
9. H.B. Huntington, *Solid State Physics-V-7*, edited by Frederick Seitz and David Turnbull (Academic Press, New York, 1958), Chapter 3.
10. O.L. Anderson, J. Phys. Chem. Solids, **24**, 909 (1963).
11. J.J. Gilvarry, Phys. Rev., **103**, 1701 (1958).
12. J.S. Thorp, M.D. Hossain, L.J.C. Bluck, T.G. Bushell. J. Mat. Sci., **15**, 903 (1980).