

ACCURATE MEASUREMENT OF MASS

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Mass is one of the base units of International System (SI) of measurement. Its accurate determination is therefore of considerable significance. Some relevant aspects of accurate measurement of mass have been enumerated. The procedure adopted for accurate comparison of masses has been elaborately described. The results of comparison of 30 pieces of a set of masses have been reported along with a measure of uncertainty of results. A sample certificate of authentication usually issued, as a result of these measurement, has also been included. The paper is rather elementary but it should pave way for further publications on similar topics.

INTRODUCTION

Internationally recognised base unit of mass kilogram (kg), an artifact made of 90% Platinum 10 Iridium, preserved at Sèvres near Paris under direct control of International Bureau of Weights and Measures (BIPM), has already been described quite elaborately [1]. The description of mode of dissemination and the measurement system involved was purposely left out to form the subject matter of a later publication.

In view of the above, the present article endeavours not only to deal with the missing links in the hierarchy of mass measurement but it would also throw light on the influence of derived quantities volume (m^3) and density (kg/m^3) on the measurement of mass with highest possible accuracy. Density of the material from which a kilogram is manufactured as well as volume of a kilogram have considerable significance in accurate comparison of masses.

Accurate measurement of volume and density is otherwise also important both from commercial and scientific standpoint. Many commodities are sold by volume in local and international markets warranting special care in its measurement. Density measurement can also reveal true and exact composition of a large number of substances. Measurement of density and volume would be dealt with in a later publication.

NATIONAL PHYSICAL STANDARD LABORATORY AND MEASUREMENT ACCURACY

"At the highest level there is no alternative source, without reinventing one or more National Bureaus of Standards, since only one agency can have custody of the

national physical reference standard for a given quantity. Any other approach could be equivalent to a decision that we should have two supreme courts".

The above quotation from the Report NBSIR 75-949 published by National Bureau of Standards, USA in July 1977 justifies, with an easily understandable example, the authority vested in the NPSL to function as the supreme focal point as far as measurement, their accuracy and uncertainty are concerned. This warrants extreme care in providing and disseminating measurement data services to all organizations interested and dependent on accurate measurement to establish credibility of their products together with educating and monitoring measurement practices at all the lower levels so that these could ultimately trace to the internationally recognised standards maintained by the National Physical and Standards Laboratory.

Set procedures are adopted to achieve the above aims and some of these are given below in case of comparison of masses indicating sources of systematic error and relevant correction.

RELATIONSHIP BETWEEN MASS, VOLUME AND DENSITY WITH CONSEQUENT IMPLICATIONS

Density (D) of a substance equals the mass (M) per unit volume (V) i.e. $D = M/V$ and $M = VD$.

It follows that mass measurement/comparison shall definitely be influenced by volume and material density of the reference mass (standard) and the test mass.

The ideal situation to avoid uncertainty/error in mass measurement/comparison would be to manufacture masses of similar denomination from materials having same density

resulting in equal volumes. This proposition is of course not practicable as weights made for achieving best accuracy, durability, uniformity e.g. reference and prototype weights are usually made of very costly and precious metals or their alloys whereas secondary, working and commercial grade weights are made from comparatively cheaper materials. Intercomparison of masses would therefore always involve corrections due to buoyancy relative to air density present in the enclosure of the balance at the time of such exercises.

DOUBLE TRANSPOSITION

It is obvious that measurement accuracy of any quantity is fundamentally dependent on the equipment involved for the purpose. Measurement/comparison of weight is no exception. The operator should first examine the balance and calibrate it in accordance with the level of desired accuracy. Present balances are fairly accurate and can cater to the needs of accuracy of mass measurement quite adequately.

In order to further ensure minimum uncertainty and to avoid most of the errors due to minor defects in the balance the standard weight 'A' and the test weight 'B' are interchanged in the pans according to the scheme given below usually known as double transposition:

Left pan	Right pan
A	B
A	B + s
A + s	B
B	A
B	A + s
B + s	A
A	B

The sensitivity weight *s* is a small weight in mg depending on the nominal accuracy of the balance. It is used to determine the sensitivity of the balance during each weighing as it may differ for weights of various denominations, as shown in Table 1.

Table 1. Change in sensitivity of the balance with change of weight.

S. No.	Denomination	Sensitivity (mg/division)
1.	500 g	82 ± 2.0
2.	1 kg	90 ± 2.4
3.	2 kg	87 ± 4.4
4.	*2 kg	90 ± 5.0
5.	5 kg	106 ± 2.4
6.	10 kg	137 ± 5.0
7.	20 kg	162 ± 2.0

Rest point of the balance with empty pans is also determined in the beginning, at the end and before any interchange of weights being compared.

CALCULATIONS

Sample calculations in case of an actual calibration of a 1 kg working standard weight made of an alloy of copper 87.5-88.5%, tin 9.5-10.5% and zinc 1.5-2.5% (test weight B) against a 1 kg secondary standard weight cast from steel of density equal to 8 kg/dm³ (standard weight A) using a secondary two pan balance of 20 kg capacity are given below to elaborate the complete procedure.

The experiment was performed at a room temperature of 20 ± 2°C and 50-60% relative humidity. In view of comparatively lower accuracy considerations, no air buoyancy corrections were employed in this particular comparison of masses.

Scale Readings		
	Left Swing	Right Swing
(i) Empty pan		
	11.3	28.6
	11.5	28.4
	11.7	28.2
	11.9	—
mean	11.6	28.4
rest point = 20.0		
(ii) Loaded		
	A + 30 mg	B
	12.2	27.8
	12.4	27.6
	12.6	27.4
	12.8	—
mean	12.5	27.6
rest point = 20.1		
(iii) Sensitivity I		
	A + 30 mg + s (500 mg)	B
	17.7	33.4
	17.8	33.2
	17.9	33.0
	18.0	—
mean	17.8	33.2
rest point = 25.5		

Rest points difference in (ii) and (iii) = 5.4
 1 Scale Division = $510/5.4 = 92.6$ mg/division

(iv) Sensitivity II		
	A + 30 mg	B + s (500 mg)
	5.0	23.6
	5.2	23.4
	5.4	23.2
	5.6	--
mean	5.3	23.4
	rest point = 14.4	

Rest point difference in (ii) and (iv) = 5.7
 1 Scale Division = $500/5.7 = 87.7$ mg/division

(v) Empty pan		
	B + 30 mg	A
	11.8	28.2
	12.0	28.0
	12.2	27.8
	12.4	--
mean	12.1	28.0
	rest point = 20.1	

(vi) Loaded with weights interchanged		
	B + 30 mg	A
	14.8	24.9
	15.0	24.7
	15.2	24.5
	15.5	--
mean	15.1	24.7
	rest point = 19.9	

(vii) Sensitivity III		
	B + 30 mg + s (500 mg)	A
	18.3	32.9
	18.4	32.7
	18.5	32.5
	18.6	--
mean	18.4	32.7

Rest point difference in (vi) and (vii) = 5.7
 1 Scale Division = $500/5.7 = 87.7$ mg/division

(viii) Sensitivity IV		
	B + 30 mg	A + s (500 mg)
	4.4	24.1
	4.6	24.0
	4.9	23.9
	5.3	--
mean	4.8	24.0
	rest point = 14.4	

Rest point difference in (vi) and (viii) = 5.5
 1 Scale Division = $500/5.5 = 90.9$ mg/division

(ix) Empty pan		
	B + 30 mg	A
	10.2	29.6
	10.4	29.4
	10.6	29.2
	10.8	--
mean	10.5	29.4
	rest point = 20.0	

Average unloaded rest points = 20.0 (1)
 Average Sensitivity = 89.7 ± 2.4 mg/division (2)

Load 'A' in left pan
 rest point = 20.1 (3)

Difference in (1) and (3) = $20.1 - 20.0 = 0.1$
 Correction to be applied = $0.1 \times 89.7 = 9$ mg
 First actual weight of 'B' = $1,000$ g + 30 mg - 9 mg
 = $1,000.0210$ g (4)

Load 'A' in right pan
 rest point = 19.9 (5)

Difference in (1) and (5) = $20.0 - 19.9 = 0.1$
 Correction to be applied = $0.1 \times 89.7 = 8.97 = 9$ mg
 Second actual weight of 'B' = $1,000.00$ g - 30 mg - 9 mg
 = 999.9610 g (6)

Calculated weight of 'B' i.e. average of equation 4 and 6 = 999.9910 g.

AUTHENTICATION CERTIFICATE

After three similar calibrations of all weights upto 500 g, a copy of the certificate prepared for the Director

Labour, Islamabad is reproduced below. The weights less than 500 g are calibrated using a single pan Sartorius balance of 200 g capacity with built-in weights by direct comparison with standard weights of the same denomination.

Calibration of a complete working standard weight kit (30 pieces) thus consumes nearly 60 man hours which is only a small indication of the complexity of accurate calibration of weights for effective and efficient traceability of commercial weights to the national standards.

Certificate of authentication of a set of working standard weights contained in two boxes marked S.W.2 for
Director of Labour, Islamabad.

Denomination	Value in grams	Standard deviation
20 kg	19,999.863	± 0.0302
10 kg	9,999.904	± 0.0774
5 kg	5,000.073	± 0.0199
2 kg	2,000.053	± 0.0065
2* kg	1,999.979	± 0.0181
1 kg	999.994	± 0.0052
500 g	500.019	± 0.0088
200 g	200.0017	± 0.00005
200* g	199.9997	± 0.00004
100 g	99.9999	± 0.00004
50 g	49.9996	± 0.00000
20 g	20.0002	± 0.00005
20* g	19.9999	± 0.00008
10 g	10.0001	± 0.00007
5 g	4.9993	± 0.00028
2 g	1.9996	± 0.00005
2* g	1.9998	± 0.00005
1 g	0.9999	± 0.00005
500 mg	0.5004	± 0.00007
200 mg	0.2002	± 0.00005
200* mg	0.1993	± 0.00004
100 mg	0.0998	± 0.00005
50 mg	0.0499	± 0.00005
20 mg	0.0199	± 0.0002
20* mg	0.0199	± 0.00005
10 mg	0.0099	± 0.00005
5 mg	0.0052	± 0.00004
2 mg	0.0022	± 0.00011
2* mg	0.0021	± 0.00009
1 mg	0.0014	± 0.00005

CHECKING OF THE BALANCE BEFORE USE

Relevant extracts from a certificate of measurement about a two pan Oertling Ltd. manufactured balance with a

capacity of 200 g issued by National Physical Laboratory, England is reproduced below in order to indicate the type of study usually carried out on a balance before actually using it for measurement/comparison of weights.

(a) *Stability of the Rest point.* The stability of the rest point was investigated over a period of time under the conditions of temperature and pressure prevailing from day to day. Daily observations were made with the balance unloaded throughout. The test occupied 13 days during which the temperature ranged from 19.8°C to 21.2°C and the pressure ranged from 993 mb to 1028 mb. The rest point varied by 1.7 divisions during this time.

(b) *Sensitivity.* The reciprocal of the sensitivity was measured under three conditions of loading as shown in the table below. Each value given in the second column represents the mass which must be added to one pan of the balance in order to change the rest point by one division of the scale. The reported measurements were carried out at temperature close to 21.5°C.

Mass on each pan (g)	Milligrams per scale division
0	0.50
100	0.49
200	0.48

(c) *Relative Lengths of the Arms.* The relative lengths of the left and right arms of the balance were deduced by comparing the rest point in the unloaded conditions with the rest point when the pans were loaded equally. The latter rest point was taken to be the mean of two rest points observed before and after interchange of the loads on the pans. With 200 g on each pan the difference between the rest points for the unloaded and loaded conditions was 0.85 division (0.41 mg) at about 23°C indicating that the left arm was shorter than the right by about 2 parts in 19,000,000. A similar measurement using a load of 100 g gave a closely similar figure for the difference in arm length.

(d) *Consistency of Performance.* The consistency of the balance was examined in two different ways.

In the first test the rest-point for a given condition of loading was observed when repeatedly releasing the balance and arresting it over a prolonged period. With the balance case kept closed a sequence of 20 rest-points was observed under each of three conditions namely in the unloaded condition, with 100 g on each pan and with 200 g on each pan.

For the unloaded condition the maximum difference between any two consecutive rest points in a sequence of twenty was 0.1 division (50 µg) and with both a 100 g load

and a 200 g load all twenty consecutive rest-points were identical.

In the second test twelve consecutive rest-points were observed under two conditions of loading, namely with 100 g on each pan and with 200 g on each pan, the load being removed and replaced between successive rest-point observations. The range of each of the four groups of three consecutive rest-points was calculated. For a load of 100 g the largest such range was 0.1 division (49 μg) and the mean range was 0.025 division (12 μg). For a load of 200 g the twelve rest-points were identical.

(e) *General Observations.* The performance of the balance is comfortably within the specification issued by Oertling Ltd. for the 200 g Projection Reading Inspectors Balance Model 0181. It was found that some of the balance components were made of slightly magnetic material namely (a) the poising nuts and studding on the beam (b) damping cylinders and (c) screw-heads in the base of the weighing chamber. The possible effect of these should be taken into account if materials likely to be magnetic are weighed.

Note. "The sensitivity of the balance for different loads and the relative lengths of arms should be redetermined by the user when the balance is reassembled."

TYPE OF BALANCES FOR REFERENCE STANDARD WEIGHTS

a) *Point-to-light Balances.* These balances are fitted with lamp and scale arrangement. Condensed light falling on the beam is reflected on a mounted scale at a distance of about 10 metres in the shape of circular spot. The beam deflection is measured through a central crosswire in the spot. The consequent magnification and the fact that light is reflected through a distance twice as much as the beam deflection considerably increases the measurement accuracy and sensitivity of the balance.

b) *Remote Control Balance.* Remote control balances for all ranges of mass measurement are now avail-

able. Transposition and addition of weights is effected from a distance of 4 to 6 metres. Beam deflection is sensed by a photocell and readings are reflected on to a scale in front of the operator. Accuracies 1 in 10^9 or above have been achieved with these balances. These balances are normally used for calibration of weights against the national kilogram.

c) *Single Pan Balances.* Sartorius balances with an accuracy of 1 in 10^6 or more are now available. When used after careful calibration and measurement of zero error before, in between and after mass measurement, these balances are quite efficient and rapid in measurement/comparison of small weights in general and particularly for fractional weights. Buoyancy correction shall however have to be made in case the weights being compared are manufactured from materials having different densities and volume from the inbuilt weights in these balances.

LABORATORY FACILITIES

All balances used for reference level work are fixed on a platform specially constructed and separated from the rest of the floor to avoid errors on account of vibrations and other disturbances of minute nature. (b) Measurement/calibration of weights is undertaken under controlled conditions of temperature ($20 \pm 1^\circ\text{C}$), and relative humidity (50 ± 5 %). A permanent record of temperature, humidity and atmospheric pressure is always kept. Air pressure measurement helps in buoyancy correction as explained earlier.

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REFERENCE

1. A. Hamid, Base Units of International System of Measurement (National Physical and Standards Laboratory, Islamabad, 1979), Bulletin, p.4.