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CALCULATION OF MONTHLY AVERAGE SOLAR RADIATION ON INCLINED SURFACES

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In this paper monthly average daily radiations incident on inclined surfaces facing directly towards the equator have been estimated. The ground reflectance of 0.23 for semidesert land of Bahawalpur (lat. 30°N) has been used.

INTRODUCTION

It is of great importance in solar energy technology to know the total radiation falling on an inclined plane. The heat energy from the Sun that reaches the surface of the earth is used at many places for the heating of water, desalination of salt or brackish water, direct conversion to electricity, and for cooking. In most cases where the maximum energy collection is required the collectors are pointed towards the Sun. In some applications, such as water heating, the flat-plate absorbing surface is mounted at an angle varying from zero to 90 degrees to the horizontal facing towards the equator. The optimum angle of tilt depends upon the latitude of the place of installation. This inclination can be selected to give equal energy absorption during the Winter and Summer.

A knowledge is therefore necessary of the amount of solar energy that will be collected by the inclined surface of the collector.

ESTIMATING OF MONTHLY AVERAGE DAILY INSOLATION ON TILTED SURFACES

To use horizontal total radiation data to estimate radiation on the tilted plane of a collector of fixed orientation, it is necessary to know $\bar{R}$, defined as

$$\bar{R} = \frac{\text{monthly average daily radiation on tilted surface}}{\text{monthly average daily radiation on a horizontal surface.}}$$

$$= \bar{H}_T / \bar{H}_h \qquad (1)$$

The radiation on a tilted surface is made up of three components, beam radiation, diffuse solar radiation, and solar radiation diffusely reflected from the ground. Therefore $\bar{R}$ can be estimated by individually considering the beam, diffuse, and reflected components of the radiation incidence on the tilted surface. Assuming diffuse and reflected radiation to be isotropic, i.e., uniformly distributed over the sky, Klein [1] has proposed that $\bar{R}$ can be expressed

$$\bar{R} = \bar{H}_T / \bar{H}_h = (1 - \bar{H}_{dh} / \bar{H}_h)\, \bar{R}_b + \frac{\bar{H}_{dh}}{\bar{H}_h} \left(\frac{1 + \cos\beta}{2}\right) + \rho \left(\frac{1 - \cos\beta}{2}\right) \qquad (2)$$

where

$\bar{H}_{dh}$ = monthly average daily diffuse radiation

$\bar{R}_b$ = ratio of the montly average daily beam radiation on the tilted surface to that on a horizontal surface for each month.

β = tilt of the surface from horizontal.

ρ = ground reflectance = 0.23, for semi desert [2].

$\bar{R}_b$ is a function of the transmittance of the atmosphere. For surfaces sloped toward the equator in the northern hemisphere

$$\bar{R}_b = \frac{\cos(\phi - \beta) \cos\delta \sin w_s' + (\Pi / 180)\, w_s' \sin(\phi - \beta) \sin\delta}{\cos\phi \cos\delta \sin w_s + (\pi/180)\, w_s \sin\phi \sin\delta} \qquad (3)$$

where ϕ is the latitude (for Bahawalpur, ϕ = 30°N), δ is the solar declination which can be approximately expressed [3].

$$\delta = 23.5^\circ \sin 360\,(284+n)/365 \qquad (4)$$

and w_s' is the sunset hour angle for the tilted surface for the mean day of the month, which is given by

$$w_s' = \min \left\{ \begin{array}{l} \cos^{-1}(-\tan\phi \tan\delta), \\ \cos^{-1}(-\tan(\phi - \beta) \tan\delta) \end{array} \right\} \qquad (5)$$

Table 1. Recommended average day for each month[1].

Month	Date	n, Day of year	δ, declination
Jan	17	17	– 20.9
Feb	16	47	– 13.0
Mar	16	75	– 2.4
Apr	15	105	9.4
May	15	135	18.8
Jun	11	162	23.1
Jul	17	198	21.2
Aug	16	228	13.5
Sep	15	258	2.2
Oct	15	288	– 9.6
Nov	14	318	– 18.9
Dec	10	344	– 23.0

Table 2. Monthly Average daily radiation on inclined surfaces at Bahawalpur.

Month	H_h MJ/m²	H_{oh} MJ/m²	H_{dh}/H_{oh}	R_b β/30°	R_b 60°	R_b 90°	H_T,MJ/m² 30°	H_T,MJ/m² 60°	H_T,MJ/m² 90°
Jan.	12.5	21.1	0.33	1.66	1.88	1.61	17.9	19.6	16.9
Feb.	16.6	25.6	0.27	1.43	1.48	1.15	21.8	22.3	18.1
Mar	20.8	31.3	0.24	1.20	1.08	0.69	23.9	22.0	15.8
Apr.	23.7	36.4	0.27	1.0	0.74	0.31	23.6	19.0	11.3
May	25.5	39.6	0.28	0.87	0.54	0.12	23.0	16.8	8.7
June	26.6	40.7	0.27	0.81	0.45	0.06	22.8	15.7	7.9
July	23.1	40.2	0.34	0.84	0.49	0.09	20.5	14.7	7.9
Aug.	21.5	37.6	0.36	0.94	0.65	0.22	20.5	16.0	9.4
Sept.	22.8	33.3	0.22	1.12	0.93	0.52	24.9	21.6	14.4
Oct.	16.4	27.7	0.31	1.35	1.34	0.98	20.3	19.9	15.5
Nov.	15.2	22.7	0.22	1.60	1.77	1.48	23.0	24.3	21.0
Dec.	11.6	19.8	0.33	1.74	2.01	1.76	17.3	19.2	16.9

"min" means the smaller of the two items in the bracket, w_s is the sunset hour angle, in degrees

$$\cos w_s = -\tan \phi \ \tan \delta \qquad (6)$$

In equation (4), n is the day of the year given for each month in Table 1. The average day is that day which has the extraterrestrial radiation nearly the same as the monthly mean value.

The ratio $\bar{H}_{dh}/\bar{H}_h$ in Equation (2) has been estimated using relationship given by Page[4].

$$\bar{H}_{dh} / \bar{H}_h = 1.00 - 1.13 \ \bar{H}_h / \bar{H}_{oh} \qquad (7)$$

where $\bar{H}_{oh}$ is the monthly average extraterrestrial radiation on horizontal surface. For these calculations value of $\bar{H}_{oh}$ have been taken from Ref [4]. $\bar{H}_h$ is the measured value of monthly average daily radiation on a horizontal surface at Bahawalpur.

The results for the 12 months of monthly average total radiation falling on tiled surfaces of various inclinations are shown in Table 2. Energy quantities are in megajoules per square meter. The effects of sloping the receiving surface to the south on the average radiation (and thus on the total radiation through the winter season) are large indeed.

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