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DISPERSION OF A SOLUTE IN MAGNETOHYDRODYNAMIC CHANNEL FLOWS

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Abstract. The dispersion of a solute in magnetohydrodynamic channel and Couette flows are considered. The effective Taylor diffusivities of the solute are calculated. It is observed that in both cases the effective diffusivity is a decreasing function of the Hartmann number.

Taylor[1–3] in a series of papers discussed the dispersion of soluble matter in the viscous, incompressible laminar flow of a fluid in a circular pipe. The case of the dispersion of a solute in non-Newtonian fluid flow in a circular pipe was discused by Fan and Hwang.[4] Soundalgekar[5] considered the effect of couple stresses on the dispersion of a solute in a channel flow. Recently Ahmadi[6] studied the dispersion of a solute in a micropolar pipe flow. Dispersion of solute in MHD flows have been considered by Gupta and Chatterjee and more recently by Soundalgekar.[7,8]

In the present work the dispersion of solute matter in magnetohydrodynamic channel and Couette flows with vanishing electric field (that is short circuit) are considered. The effective Taylor diffusivities of the solute are calculated. It is observed that the effective diffusivities are decreasing functions of the Hartmann number. The asymptotic forms of the Taylor diffusivities for small and large Hartmann numbers are also obtained and discussed.

Basic Equation. The concentration c of the solute diffusing in a fluid flowing in a channel satisfies the following equation:

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right) \tag{1}$$

where D is the molecular diffusion coefficient, u is the velocity distribution, x, y are the space coordinates in the direction of the flow and normal to it, respectively, and t is the time. We assume that the channel is wide enough so that the dependence on the third space coordinate can be ignored.

Following Taylor[1–3] we assume that the diffusion in the flow direction is much smaller than the radial diffusion, i.e.

$$\frac{\partial^2 c}{\partial x^2} << \frac{\partial^2 c}{\partial y^2} \tag{2}$$

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Employing now the following dimensionless quantities

$$\xi = \frac{x-\bar{u}t}{L}, \; \eta = y/h, \; \theta = \frac{tL}{\bar{u}} \tag{3}$$

The diffusion equation (1) in a frame moving with the average velocity $\bar{u}$ becomes

$$\frac{\bar{u}}{L}\frac{\partial c}{\partial \theta} + \frac{w}{L}\frac{\partial c}{\partial \xi} = \frac{D}{h^2}\frac{\partial^2 c}{\partial \eta^2} \tag{4}$$

where $w = u-\bar{u}$, h (5) is the half width of the channel and L is a given length along the flow direction.

If we assume that the Taylor limiting condition to be valid, then the partial equilibrium may be assumed in any cross-section of the channel and hence $\partial c/\partial \theta$ is negligible. Therefore, c satisfies the following equation:

$$\frac{\partial^2 c}{\partial n^2} = \frac{h^2}{DL} w \frac{\partial c}{\partial \xi} \tag{6}$$

For given $w(\eta)$ the effective Taylor diffusivity may be calculated by the method which we will outline in the specific cases.

Magnetohydrodynamic Channel Flow. The expression for the velocity in a fully developed flow of a viscous, incompressible, electrically conducting fluid between two parallel walls and under a constant transverse magnetic field and in the absence of electric field as given by Pai,[9] Cambel[10] and Sutton and Sherman[11] is

$$\frac{u}{u_0} = \frac{\cosh M - \cosh (M\eta)}{\cosh M - 1} \tag{7}$$

where u_0 is the maximum velocity

$$u_0 = -\frac{h^2}{\mu}\frac{\partial p}{\partial x}\frac{\cosh M - 1}{M^2 \cosh M}$$

$\eta = y/h =$ dimensionless transverse coordinate; $M =$ Hartmann number

$\partial p/\partial x$ is the pressure drop and μ is the viscosity of fluid. The average velocity $\bar{u}$ is

$$\frac{\bar{u}}{u_0} = \frac{M \cosh M - \sinh M}{M (\cosh M - 1)} \tag{8}$$

It must be pointed out that the velocity distribution (7) is for the short circuit condition and is quite different from the ones employed in (7) and (8).
The relative velocity then becomes

$$\frac{w}{u_0} = \frac{u - \bar{u}}{u_0} = \frac{\sinh M - M \cosh (M\eta)}{M (\cosh M - 1)} \tag{9}$$

Substituting for w from (9) into (5) and integrating twice under the assumption of constant $\partial c/\partial \xi$, we find

$$c = \frac{h^2 u_0}{2DL} \frac{\eta^2 M \sinh M - 2 \cosh (M\eta)}{M^2 (\cosh M - 1)} \cdot \frac{\partial c}{\partial \xi} + c_0 \tag{10}$$

where we have used the boundary conditions

$$\left. \frac{\partial c}{\partial \eta} \right|_{\eta = \pm 1} = 0 \tag{11}$$

and c_0 is a constant which can be determined from the entry condition.

Now the volume rate of the transport of the solute across a section of the channel is given by

$$Q = \int_{-h}^{h} c\, w\, dy = h \int_{-1}^{1} c\, \nu\, d\eta$$

$$= \frac{h^3 u_0^2}{2DL} \frac{[2M^2 + 3M \sinh 2M - (8 + \frac{4}{3} M^2) \sinh^2 M]}{M^4 (\cosh M - 1)^2} \frac{\partial c}{\partial \xi} \tag{12}$$

On comparing (12) with Fick's law of diffusion

$$Q = -2hD^* \frac{\partial c}{\partial x} \tag{13}$$

we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective Taylor diffusion coefficient D^* given by

$$D^* = \frac{h^2 u_0^2 F(M)}{D} \tag{14}$$

where

$$F(M) = \frac{(8 + \frac{4}{3} M^2) \sinh^2 M - 2M^2 - 3M \sinh 2M}{4M^4 (\cosh M - 1)^2} \tag{15}$$

Figure 1 shows the variation of F with Hartmann number.
For small Hartman number the above becomes

$$F(M) = (9/945) [1 - M/30 + \ldots] \tag{16}$$

which reduces to that of a simple viscous fluid for $M = 0$. For large M, the expression (15) takes the following asymtotic form.

$$F_a(M) = (1/3M^2) (1 - 9/2M + 12/M^2 + \ldots) \tag{17}$$

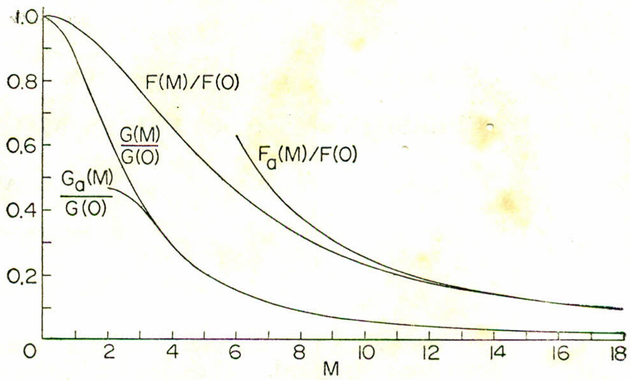

Fig. 1. The variation of the functions F (M) and G(M) and their asymptotic forms with Hartmann number M.

From Fig. 1 and equations (14–17) we conclude that for fixed u_0 the effective diffusivity D^* decreases with an increase in the Hartmann number and the rate of decay is proportional to $1/M^2$ for very large Hartmann number. From Fig. 1 it is also observed that for M greater than 11, F_a becomes almost identical to F.

Magnetohydrodynamic Couette Flow. The expression for a viscous, incompressible, electrically conducting fluid in a Couette flow under a constant transverse magnetic field with zero electric field (that is short circuit conditions) is given by [9–12]

$$\frac{u}{\nu} = \frac{\sinh M\eta}{\sinh M} \tag{18}$$

where $\eta = \pm 1$ are the boundaries of the flow and ν is the velocity of boundary plates. The average velocity is of course zero and hence

$$w = u = \nu \frac{\sinh M\eta}{\sinh M} \tag{19}$$

Employing the above in (6) and integration twice and making use of the boundary conditions (11) we find

$$c = \frac{h^2 \nu}{DL} \frac{\sinh M\eta - \eta M \cosh M}{M^2 \sin M} \frac{\partial c}{\partial \xi} + c_0 \tag{20}$$

The volume rate of transport of the solute across a section of the channel is then given by

$$Q = \frac{h^2 \nu^2}{2DL} \frac{3 \sinh 2M - 2M \cosh 2M - 4M}{M^3 \sinh^2 M} \frac{\partial c}{\partial \xi} \tag{21}$$

On comparing the above with (13) we find

$$D^* = \frac{h^2 \nu^2}{D} G(M) \tag{22}$$

where

$$G(M) = \frac{2(M) \cosh 2M + 4M - 3 \sinh 2M}{4M^3 \sinh^2 M} \tag{23}$$

The expression (22) is the effective Taylor diffusivity for MHD Couette flow. Figure 1 shows the variation of G with Hartmann number.

For small values of M we find

$$G(M) = \frac{2}{15}\left(1 - \frac{M^2}{7} + \dots\right) \qquad (24)$$
$$M \to 0$$

For $M = 0$, $G(M) = 2/15$ which is the special case of a simple viscous fluid.

For large values of M the asymptotic form of $G(M)$ becomes

$$G_a(M) = \frac{1}{M^2}\left(1 - \frac{3}{2M} + \dots\right) \qquad (25)$$
$$M \to \infty$$

The asymptotic value G_a is also being plotted in Fig. 1. It is observed that for M greater than 3 the magnitude of G_a is identical to that of G. From equation (25) we conclude that the effective diffusivity decays to zero as $1/M^2$ for large M.

Conclusions

The result of ref.7 and 8 is extended to the case of short circuit MHD channel and Couette flows. The effective Taylor diffusivity in both cases are found to be a strong function of the Hartmann number. For zero Hartmann number it approaches to that of a non-magnetic viscous fluid which is a positive constant. For an increase in Hartmann number the effective diffusivity decrease and for large Hartmann number flows it approaches zero as M^{-2}.

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