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MIXING OF FREE FLOWING GRANULAR MATERIALS IN HORIZONTAL-MIXERS

Part I. Effect of the Inclined Flights on Mixing Time

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Abstract. Experiments on mixing of —16+18 B.S.S. mesh sand, (coloured and uncoloured) in a horizontal rotating drum with and without inclined flights both for superimposed and adjacent-layers have been carried out. The effect of load-fraction on the mixing time also has been studied.

The results show that in *absence* of flights for superimposed layers, the mixing time is shorter as compared to adjacent parallel layers while their presence introduces axial-movement and reduces mixing time in both the cases in proportion with the slope of the flights. It has been further observed that with the increase in the load-fraction p, the mixing time t also increases. A general correlation between σ^2 and 't', of the type $\sigma^2=\frac{t}{a+bt}+c$, has been proposed.

Introduction

Mixing of powders or granular materials is perhaps as old as history of mankind. If time is no consideration, old devices or absolete methods, may well be used to reach nearly perfect mixing. But with the advent of industry, the requirements of complete and quick mixing have multiplied and so have the mixing techniques, resulting in a variety of mixing machines namely the horizontal or inclined rotating cylinders, the ribbon or spiral mixer, the cone mixer, the V-mixer and more recently the zigzag mixers. These mixers are based on varying combinations of radial and axial movement of granules. The best mixer perhaps is the one which represents an equal combination of the radial as well as axial movements. Any mixe rwhich does not bring about or induces these movements simultaneously will not be a good mixer.

In a simple horizontal rotating cylinder, concentric layers of granules rise along the periphery and then slide down the inclined plane presumably to rejoin the original layers. If the cylinder is made inclined or that inclined flights are fitted on the inner side of the shell, as has been done in this case, the motion of the granules would be different in the manner that they will first be lifted and then thrown over the inclined bed at an angle equal to the slope of the flights. If the consecutive flights are fitted in opposite direction, the lifted material will alternately move in opposite direction that is forward and backwards along the horizontal axis.

Two ideal cases have been studied namely case I where the cylinder is fed with superimposed coloured and uncoloured layers equal in weight and in case II where the granules were present in the form of adjacent parallel layers, and the effect of the load-fraction p and the slope of the flights on the mixing time has been studied.

Discussion of Earlier Work

Mixing of solid–solid particles can best be represented by taking into account both the characteristics of the mixing machine and the physical properties of the solid materials. Since there is now a variety of mixers, possibility exists that the final correlation between the proprties and the mixing time, varies from one machine to another. No serious effort appears to have been made to bring forth such a correlation. But on the other hand much stress has been laid in evolving a criterion to determine the degree of mixing and then correlating it with either the mixing time or the number of the revolution per minute of the rotating cylinder. Brothman and his coworkers[1] have developed a theory on the basis of shear mixing and claim that the mixing increases as the fresh surface between two kinds of the materials increases and reaches a maximum value in a completely mixed material. The experiments in support of their theory were presumably carried out in a rotating cylinder but no mention has been made about the use of inclined flights. Coulson and Maitra[2] though used flights in some of their experiments, to introduce axial movement of the granules but did not study the effect of inclined flights on the mixing time. Blumberg and Maritz[3] and Lacey[4] in their theories developed on a horizontal rotating cylinder (similar to Coulson and Maitra's apparatus) have expressed their results in terms of variance σ^2 as function of mixing time 't' and finally gave a diffusion theory based on Fick's law. These authors did not make use of inclined flights in their experiments to study their effect. Kaufman[5] studied mixing in a double-cone blender and a three-twin-shell blender to find that the twin-shell blender gave slightly better results and proved that greater mixing rates are achieved in larger blenders. They plotted log variance (log σ^2) against log revolutions per minute (log N) and found that the variance decreases rapidly as the mixing progresses. He also observed that during the initial stages of the mixing, no significant difference is found between the variance observed on the charges containing 40% and the other one containing 50% penicillin. Carley-Macauly and Donald[6] after a critical review of the previous work, have dealt with the mechanism of mixing process using tumbling mixers (double-cone). The results have been expressed in terms of the estimated variance $Vp = V - \sigma_R^2$ as function of number of revolutions N, in the form of $\log (Vp) = -(t/N) + c$. Rutger's[7] work on mixing is confined to an inclined rotating cylinder. A theory of longitudinal mixing based on general diffusion model has been worked out. This work too has been carried out in absence of flights.

In the present work variance σ^2 has been used to indicate the degree of mixing and the results have been first translated into variance and then plotted against the mixing time t.

Apparatus and Procedure

The apparatus is a simple 11 in dia cylinder with removable sides, rotating at 24 rev/min on a pair of rollers driven by a motor. The cylinder is filled with sand, in 10, 20, 30 and 40% of the free volume (by weight). All fillings consist of 50% each of coloured and uncoloured sand granulers, spread evenly either in the form of superimposed or adjacent parallel layers.

A L-shaped sampler used for drawing the samples may be seen in Fig. 1a. The plunger (L) is raised from its seat at the sampling end and the sample sucked in and trapped by a remote lever. In each experiment, 9 samples were collected from the positions shown in Fig. 1b at fixed intervals of time and the variance (Table 1) calculated with the help of the usual statistical equation $\sigma^2 = (x - \bar{x})^2/n$ where $\bar{x}$ is $\Sigma x/n$.

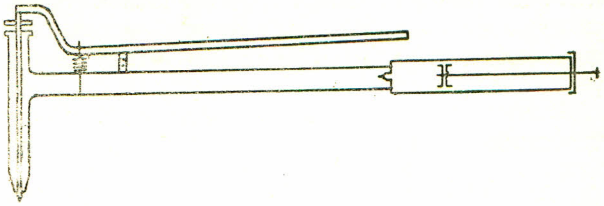

Fig. 1a. Sampler

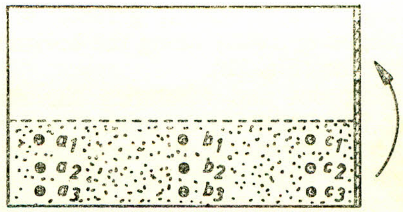

Fig. 1b. Granular bed

Results and Discussion

As already stated the present study has been restricted to two ideal cases namely when the feed is present in the form of superimposed horizontal (case I) and adjacent parallel layers (case II) where the effect of S and p on t has been noted. The introduction of inclined flights at this stage brings about axial movement of the granules in both the cases thereby accelerating the mixing process (Fig. Ic).

1. *Effect of Load-Fraction* p *in Presence of Flights*

Consider that the granules in the bed travel in concentric layers and that this pattern of granular movement remain unaffected by the load-fraction p. On emerging at the free surface of the inclined plane, the granules are likely to be displaced from the actual

TABLE 1. SAMPLE CALCULATIONS OF VARIANCE FOR ADJACENT PARALLEL LAYERS (p, 0.10; t, 1 min).

Sample	Composition (%) Coloured particles (x%)	Uncoloured particles(%)	Mean value $n_s=9$ $\Sigma(x/n)=\bar{x}$	Deviation $(x-\bar{x})$	Deviation square $(x-\bar{x})^2$	Variance. (σ^2) $n\Sigma=9(x-\bar{x}/n)^2$
I. *Without flights*						
a1	100.00	—		+51.3	2631.00	
a2	100.00	—		+51.3	2631.00	
a3	100.00	—		+51.3	2631.00	
b1	33.3	66.7		—15.4	236.00	
b2	66.3	33.7	48.7	+ 0.4	310.00	1728
b3	49.1	50.0		+17.6	0.16	
c1	—	100.00		—48.7	2372.00	
c2	—	100.00		—48.7	2372.00	
c3	—	100.00		—48.7	2372.00	
II. *With flights*						
a1	46.6	53.4		— 0.5	0.25	
a2	53.0	47.0		+ 5.9	34.90	
a3	49.6	50.4		+ 2.5	6.25	
b1	45.4	54.6		— 1.7	2.89	
b2	47.3	52.7	47.1	— 0.2	0.04	16.6
b3	39.2	60.8		— 7.9	62.32	
c1	50.0	50.0		+ 2.9	8.40	
c2	51.2	48.8		+ 4.1	16.80	
c3	42.9	57.1		— 4.2	17.60	

paths while rolling or sliding down the inclined plane. The only zone where apparently the mixing could take place, appears to be in this 'triangular-shaped' sliding layer in the inclined plane. The mixing taking place in this zone may be due to an overall effect of the combination of the following movements of the granules:

(a) Short-circuiting of the actual paths, i.e. re-entering the concentric layers at places other than the original ones (Fig. 2a).

(b) Striking the slow moving granules of the sub-layers and shooting mostly along the horizontal axis, in either direction (Fig. 2b).

(c) Emerging out and reentering the same concentric layer, i.e. continuing the original path (Fig. 2c).

Superimposed Layers. In mixing of superimposed layers, situation 1 probably dominates situation 2 which may only be of secondary nature while situation 3 appears to be rare. If situation 1 is considered to dominate, then the mixing (in the surface layers of the inclined plane) is due to the difference in velocities of the tumbling granules in different layers, i.e. the faster moving will slide over slow moving layers and reenter at points ahead of their original layers. In situation 2, the fast moving surface granules may collide with slow moving granules and shoot in either direction along the horizontal axis. The axial movement thus induced is insignificant and the mixing time tends to reach infinity. In situation 3, the granules in the sliding layers will enter the respective concentric circle they originally leave. Should this happen, no mixing will take place. It may further be added that in case of superimposed layers, situation 1 appears to prevail with a very little contribution from situation 2.

In presence of flights when p is plotted against t, the type of curves obtained are shown in Fig. 3a. The trend of these curves shows that increase in p enhances the time of mixing t. When p is increased beyond 0.4, the mixing time is presumably increased and there is no mixing for $p = 1$. Introduction of flights also reduces the time of mixing in proportionate to the angle of the slope which is apparent when the curved (without flights) is compared with a, b and c in Fig. 3a. Thus it will be seen that greater the value of S, shorter the t, with the result that for maximum slope of 45° t tends to become independent of p over a range of 0.1–0.4.

Adjacent or Parallel Layers. When the granules are present in the form of adjacent parallel layers, mixing in the cylinder will primarily be governed by situation 2, i.e. by axial movement (of horizontal diffusion) of the granules. Referring to Fig. 3b where p has been plotted against t, with and without flights, it may be seen that with the increase in p, the value of t also increases according to equation $p = k(t)^{n'}$ but when flights are introduced, it becomes linear, i.e. $n = 1$. Again when there are no flights, the relationship between p and t for superimposed and parallel layers, as plotted in Fig. 3(c), indicates that for all values of p, the ratio between the time spent on complete mixing due to axial and radial movement of the granules, A_t/R_t remains almost constant (Table 2). It shows that the radial movement of the granules is 21–24 times faster than the axial movement (diffusion) of the granules.

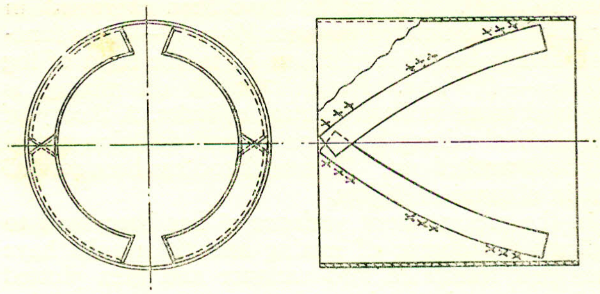

Fig. 1c. Flight position

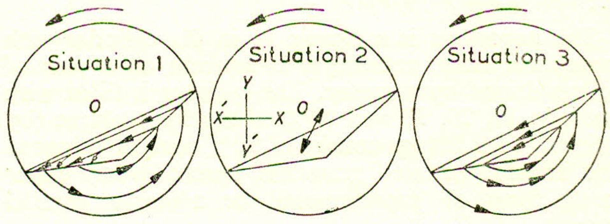

Fig. 2.

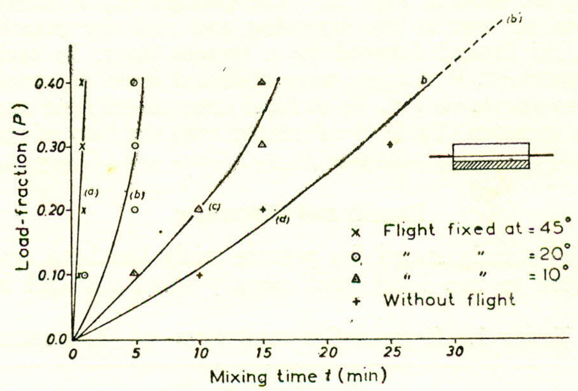

Fig. 3a. Superimposed horizontal layers-effect of flights inclination and load fraction on mixing time.

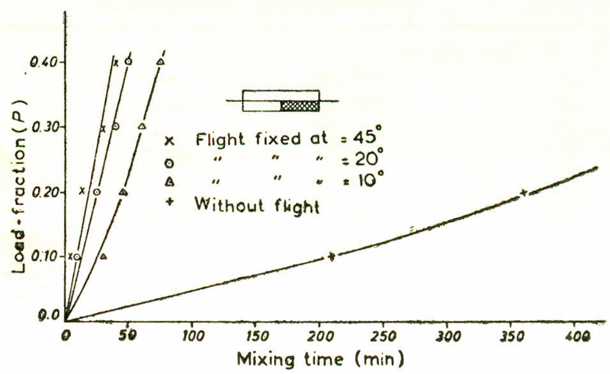

Fig. 3b. Adjacent layers-effect of flight inclination and load fraction on mixing time.

Thus it will be seen that in both the cases, i.e. superimposed and parallel layers, the value of k and n in equation $p = k(t)^{n'}$ as given in Table 3, shows that the value of n falls with the rise in the slope of the flights in accordance with the equation $n' = -k'(S)^{n''+c}$, where $p = K(t)^{-K'(S)n''+c}$

2. *Radial Movement vs Axial Movement of the Mixing Layers*

As discussed earlier, the mixing in superimposed layers is apparently due to radial movement of the granules while on the other hand the mixing in parallel layers is considered to take place through axial movement of the granules. The mixing is faster in case I while it is very slow in case II which can readily be visualised from Fig. 4 where the time spent on radial mixing is R_t plotted against the time spent on axial A_t, for a value of $p = 0.1$, 0.2, 0.3 and 0.4. R_t is linear function of A_t, i.e. $R_t = 3.63 \times 10^{-2} (A_t) + 2.363$. The fourth point plotted in the figure represents an imaginery value of A_t because when the granules remain in contact with each other for a long time, their colour gets faded to make final separation very difficult. As the value of p increases beyond 0.4, the straight-line forms an asymptote with x-axis which means that the value of A_t would tend to infinity as p approaches 1.

3. *Variance in Relation to Mixing Time* t

Superimposed Layers (*Without Flights*). Referring to Fig. 5 where σ^2 has been plotted against t, ($p = 0.1$, 0.2, 0.3 and 0.4) for different value of S, it will be seen that the type of the curves obtained is expressed by the relationship $\sigma^2 = t/(a + bt) + c$, where a, b and c are constants.

Superimposed Layers (*With Flights*). When flights are introduced, the time of mixing is shortened and is proportional to the slope of the flights, i.e. greater the slope, lesser the time. With 10, 20 and 45° slope, the set of curves for all values of p may be seen in Figs. 6, 7 and 8. At 20° and above the mixing is so rapid that the effect of p is practically eliminated, with the result that a single hyperbolic relationship for all values of p is obtained:

$$\sigma^2 = \frac{10^2 t}{4.42t - 2.48} - 0.86$$

and $$\sigma^2 = \frac{10^2 t}{17.3t - 8.3} - 5.05.$$

A comparative study of these curves indicates that a minimum value of σ^2 is quickly reached with 45° slope as compared to a slope of 10° Further the 'bend' in the rectangular hyperbola is more sharp at 45° which means that the mixing becomes rather slow as the value of S falls from 45° to 10°.

Adjacent Layers (*Without Flights*). In case of adjacent layers when no flights are used, the relationship between σ^2 and t for different value of p may be seen in Fig. 9. These curves are again represented by an equation for rectangular hyperbola. The fall in σ^2 is rather slow that is the mixing is very slow for all values of p. For example, for $p = 0.3$ and 0.4, further fall in the value of σ^2 i.e. after 1500 is very slow. For $p = 0.1$, the minimum value of σ^2 remains almost constant ($\sigma^2 = 75$) which means that even if the sample is mixed for an infinite time, complete mixing will not take place.

Adjacent Layers (*With Flights*). When flights are introduced, the results obtained may be seen in

TABLE 2. COMPARISON OF MIXING TIME FOR SUPERIMPOSED AND HORIZONTAL LAYERS (WITHOUT FLIGHTS).

Load-fraction p	Ratio A_t/R_t
0.10	$\frac{210}{10} = 21$
0.20	$\frac{360}{15} = 24$
0.30	$\frac{480}{20} = 24$
0.40	$\frac{620}{30} = 21$

TABLE 3. THE VALUES OF CONSTANTS K AND n'.

Angle of inclination of the flights (degrees) S	Superimposed horizontal layers		Adjacent parallel layers	
	k	n'	k	n'
0	$3{\cdot}64 \times 10^{-3}$	1·439	$1{\cdot}29 \times 10^{-5}$	1·65
10	$1{\cdot}159 \times 10^{-2}$	1·261	$5{\cdot}25 \times 10^{-4}$	1·55
20	$3{\cdot}910 \times 10^{-2}$	1·183	$1{\cdot}43 \times 10^{-2}$	1·00
45	$3{\cdot}00 \times 10^{-1}$	1·00	$1{\cdot}11 \times 10^{-2}$	1·00

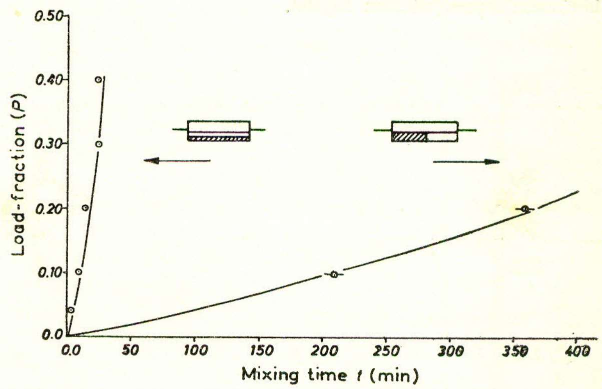

Fig. 3c. Load fraction (P) vs. mixing time (t) without flight.

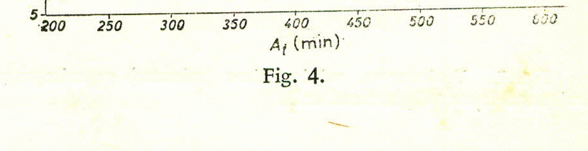

Fig. 4.

Nomenclature: a, a constant; A_t, mixing time due to axial-movement of the granules; b, a constant; c, a constant; k, a constant; n, number of samples; n, slope of a straight line; p, degrees of filling or load fraction; R_t, mixing time due to radial-movement of the granules, minutes; S, angle of inclination of the flights, degrees; t, time for complete mixing, minutes; x, number of the coloured granules, percentage; $\bar{x}$, $\Sigma \frac{x}{n}$ σ^2, variance $= \sum^{n=9} \left(\frac{x - \bar{x}}{n}\right)^2$

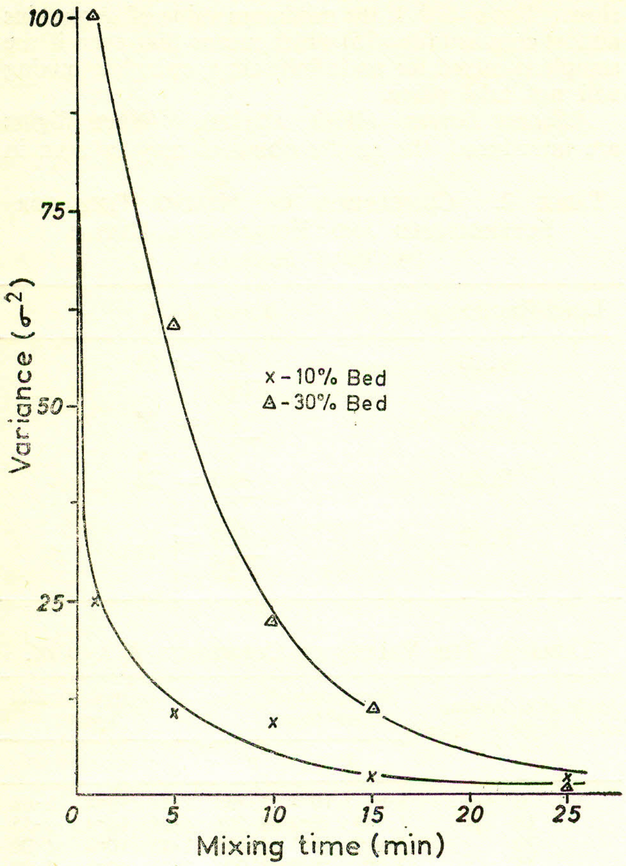

Fig. 5. Variance σ2 vs. mixing time (*t*) superimposed horizontal layers (without flights).

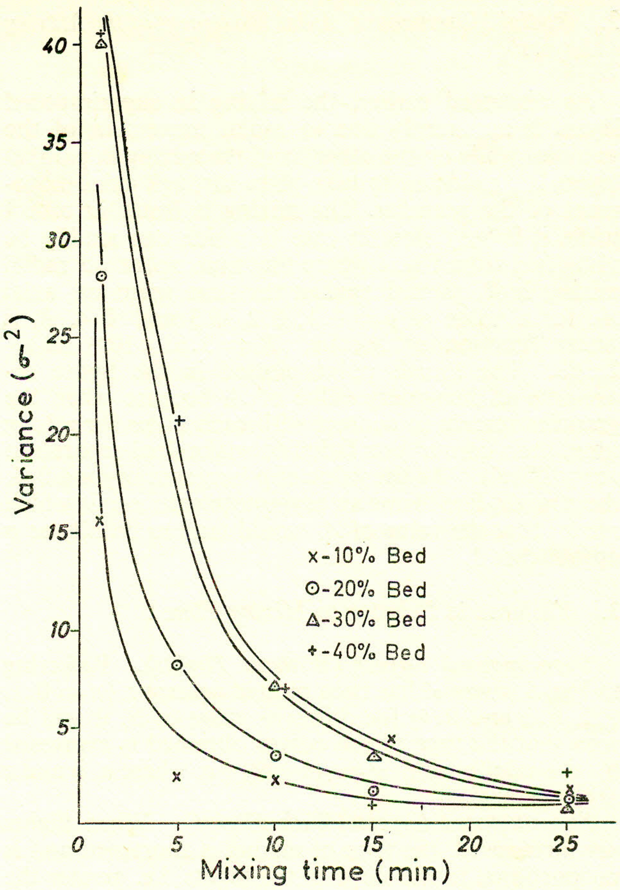

Fig. 6. Variance vs. mixing time (minutes) superimposed horizontal layers (flight fixed at 10°).

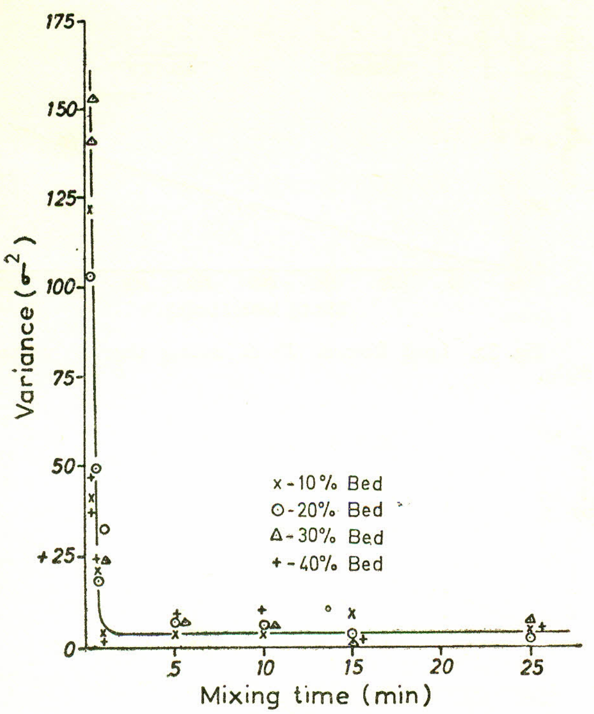

Fig. 7. Variance vs. mixing time (minutes) superimposed horizontal layers (flight fixed at 20°)

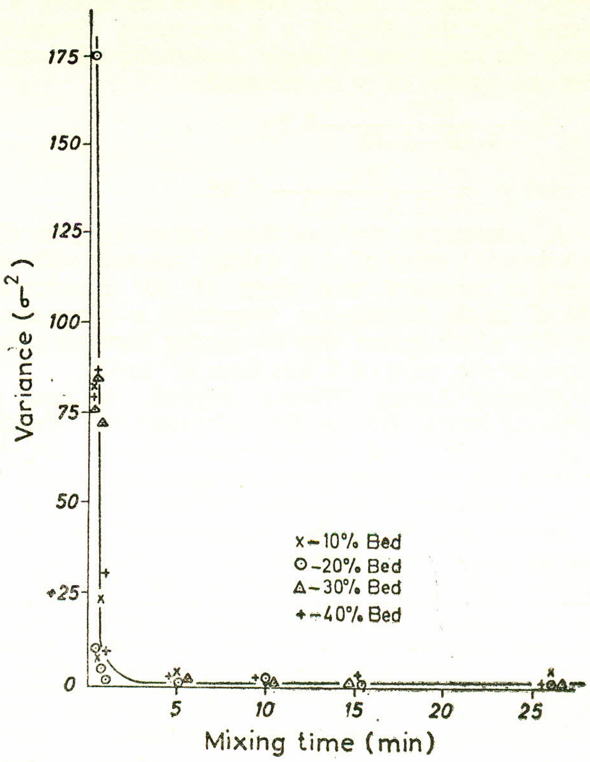

Fig. 8. Variance vs. mixing time (minutes) superimposed horizontal layers (flight fixed at 45°).

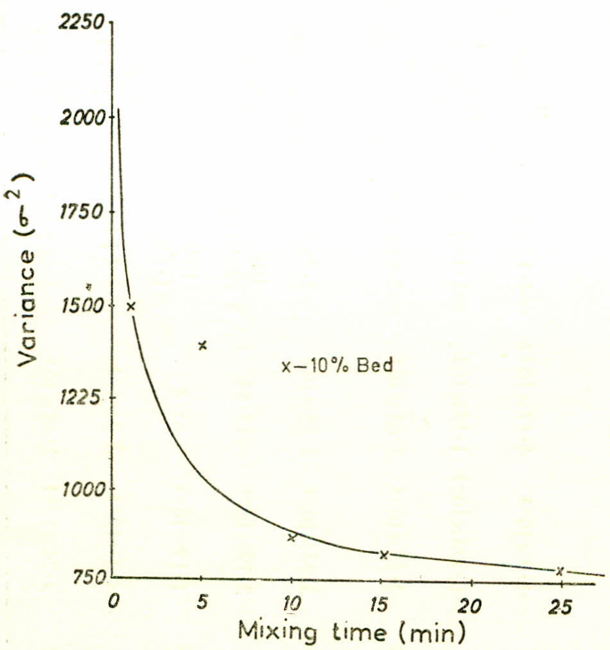

Fig. 9. Varianee vs. mixing time (minutes) adjacent layers (without fiight).

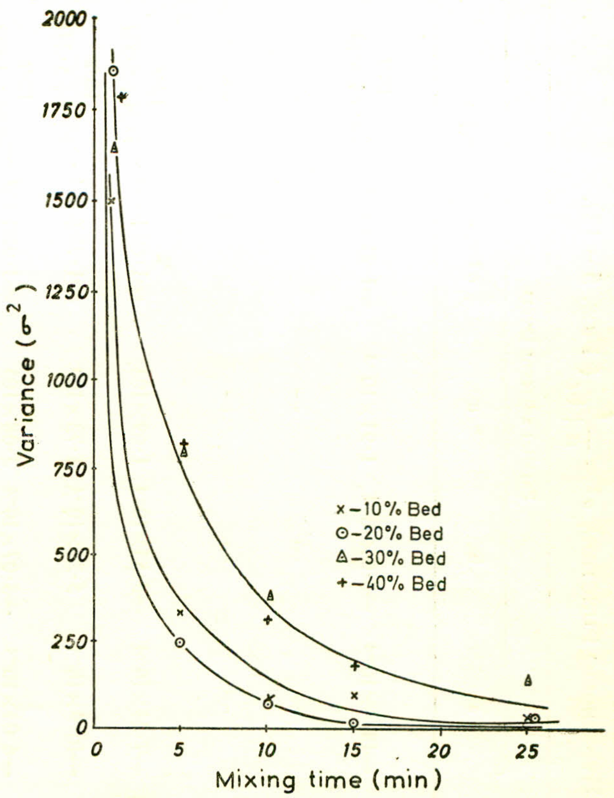

Fig. 10. Variance vs. mixing time (minutes) adjacent layers (flight fixed at 10°).

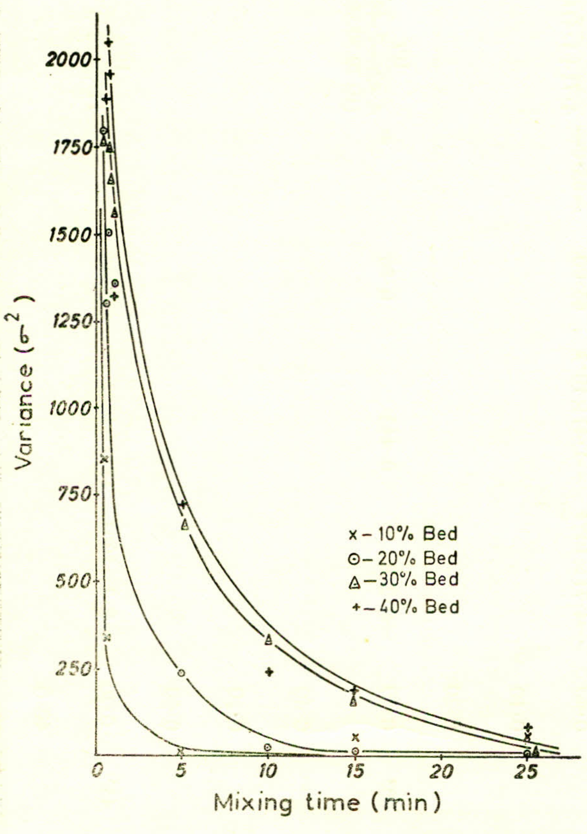

Fig. 11. Variance vs. mixing time (minutes) adjacent layers (flight fixed at 20°).

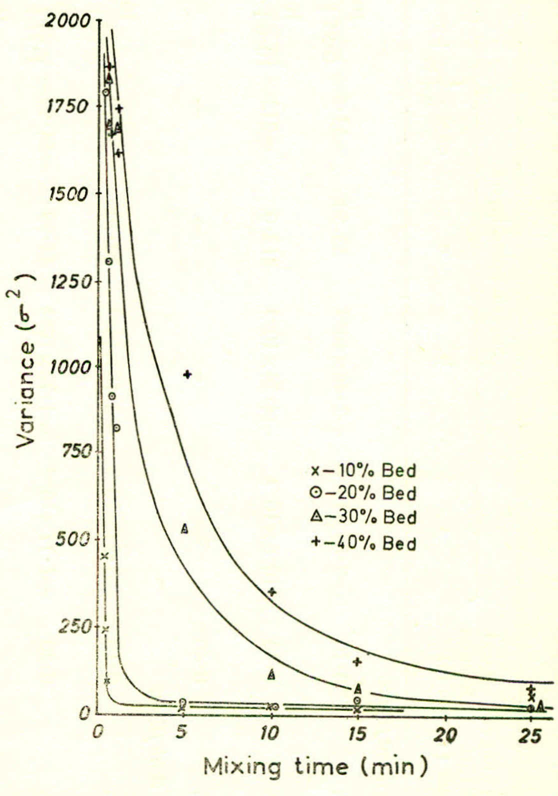

Fig. 12. Variance vs. mixing time (minutes) adjacent layers (flights fixed at 45°).

TABLE 4. VALUES OF THE CONSTANTS a,b, AND c IN RECTANGULAR HYPERBOLIC FUNCTION, $\sigma^2=\frac{t}{a+bt}$ (FIGS. 5 – 12).

Inclination of the flights degrees (s)	Load fraction (p)	Superimposed layers				Parallel adjacent layers			
		a	b	c	$\sigma2=\frac{t}{a+bt}+c$	a	b	c	$\sigma2=\frac{t}{a+abt}+c$
I. *Withont flights*									
1.	0·10	$-9{\cdot}85\times10^{-2}$	$-2{\cdot}46\times10^{-2}$	33.50	$=33{\cdot}5-\frac{10^2t}{0{\cdot}95+2{\cdot}46t}$	$-18{\cdot}4\times10^{-4}$	$-8{\cdot}0\times10^{-4}$	$1{\cdot}91\times10^{-3}$	$=1{\cdot}91+10^3-\frac{10^4t}{18{\cdot}4+8{\cdot}0t}$
2.	0·30	$-10{\cdot}6\times10^{-3}$	$-4{\cdot}24\times10^{-3}$	212·0	$=212-\frac{10^3t}{10{\cdot}6+4{\cdot}24t}$	—	—	—	—
3.	0·40	—	—	—	—	—	—	—	— —
II. *With flights*									
1.	0·10	$-3{\cdot}82\times10^{-2}$	$-3{\cdot}18\times10^{-2}$	30·70	$=30{\cdot}70-\frac{10^2t}{3{\cdot}82+3{\cdot}18t}$	$-7{\cdot}24\times10^{-4}$	$-5{\cdot}53\times10^{-4}$	$1{\cdot}69\times10^3$	$=1{\cdot}69+10^3-\frac{10^4t}{7{\cdot}24+5{\cdot}53t}$
2.	0·20	$-1{\cdot}77\times10^{-2}$	$-1{\cdot}72\times10^{-2}$	54·99	$=54{\cdot}99-\frac{10^2t}{1{\cdot}77+1{\cdot}72t}$	$-10{\cdot}9\times10^{-4}$	$5{\cdot}24\times10^{-4}$	$1{\cdot}72\times10^3$	$=1{\cdot}72+10^3-\frac{10^4t}{10{\cdot}9+5{\cdot}24t}$
3.	0·30	$-3{\cdot}92\times10^{-2}$	$-1{\cdot}40\times10^{-2}$	62·60	$=62{\cdot}60-\frac{10^2t}{3{\cdot}92+1{\cdot}40t}$	$-6{\cdot}05\times10^{-4}$	$-3{\cdot}07\times10^{-4}$	$3{\cdot}08\times10^3$	$=3{\cdot}08+10^3-\frac{10^4t}{6{\cdot}05+3{\cdot}07t}$
4.	0·40	$-4{\cdot}71\times10^{-2}$	$-1{\cdot}41\times10^{-2}$	63·00	$=63{\cdot}0-\frac{10^2t}{4{\cdot}71+1{\cdot}41t}$	—	—	—	— —
1.	0·10					-3×10^{-3}	$-1{\cdot}9\times10^{-3}$	$4{\cdot}17\times10^{+2}$	$=4{\cdot}17+10^{+2}-\frac{10^3t}{3+1{\cdot}94t}$
2.	0·20					$-1{\cdot}8\times10^{-3}$	$-0{\cdot}733\times10^{-3}$	$1{\cdot}15\times10^3$	$=1{\cdot}15+10^3-\frac{10^3t}{1{\cdot}8+0733t}$
3. 20°	0·30	$-0{\cdot}248$	0·442	0·86	$=\frac{10t}{4{\cdot}42t-2{\cdot}48}-0{\cdot}86$ (10,20,30,40)	$-7{\cdot}95\times10^{-4}$	$-3{\cdot}35\times10^{-4}$	$2{\cdot}63\times10^3$	$=2{\cdot}63+10^3-\frac{10^4t}{7{\cdot}95+3{\cdot}95t}$
4.	0·40					$-13{\cdot}7\times10^{-4}$	$3{\cdot}95\times10^{-4}$	$2{\cdot}25\times10^3$	$=2{\cdot}25+10^3-\frac{10^4t}{13{\cdot}7+3{\cdot}95t}$
1.	0·10					$-1{\cdot}7\times10^{-2}$	$3{\cdot}79\times10^{-2}$	$-12{\cdot}64$	$=\frac{10t}{3{\cdot}79t-1{\cdot}7}-12{\cdot}64$
2.	0·20					$-1{\cdot}01\times10^{-2}$	$1{\cdot}15\times10^{-2}$	$-65{\cdot}4$	$=\frac{10^2t}{=1{\cdot}15t-1{\cdot}01}-65{\cdot}4$
3. 45°	0·30	$-8{\cdot}3\times10^{-2}$	$17{\cdot}3\times10^{-2}$	$-5{\cdot}05$	$\frac{10^2t}{17{\cdot}3t-8{\cdot}3}-5{\cdot}05$	$-7{\cdot}0\times10^{-4}$	$-4{\cdot}2\times10^{-4}$	$2{\cdot}2\times10^3$	$=2{\cdot}2+10^3-\frac{10^4t}{7+4{\cdot}2t}$
4.	40·0					$-9{\cdot}66\times10^{-4}$	$3{\cdot}74\times10^{-4}$	$2{\cdot}48\times10^3$	$=2{\cdot}48+10^3-\frac{10^4t}{9{\cdot}96+3{\cdot}74t}$

Figs. 10, 11 and 12. All these figures show that there is a sharp fall proportionate to the slope of the flights for all values of *p*, i.e. complete mixing is obtained with the introduction of inclined flights and greater the slope, quicker the mixing. Again all the curves obtained are of the same nature and may, therefore, be represented by a hyperbolic equation.

Conclusion

The presence of feed in a mixer in the form of superimposed or parallel layers represents two ideal cases. In superimposed layers, the mixing is most probably due to radial movement of the granules alone without any significant contribution from the axial movement. The greater the speed, quicker the mixing. In the same manner, the mixing if at all takes place in case of parallel layers, is due to the axial movement of the granules which is limited to 'sliding layer triangle' on the inclined plane as in superimposed layers. The introduction of inclined flights brings about the axial movement in both the cases, that is to say it supplements the radial movement in superimposed layers and introduces axial movement in parallel layers. The mixing as described in the present investigation is represented by the general equation of the type $\sigma^2 = \frac{t}{a+bt} + c$ the values of the constant given in Table 4.

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