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THE E_{010} CAVITY RESONATOR, THE TEMPERATURE VARIATION OF RESONANT FREQUENCY, THE RESONANCE CURVES AND THEIR ANALYSIS

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An E_{010} circular cylindrical cavity resonator has been designed and constructed to investigate the dielectric properties of different dielectrics. Described in the paper are the temperature variation of resonant frequencies and the experimental techniques to draw the Q-curves of the air-filled and loaded cavity resonator at 10 cm wavelength band.

The shapes of the resonance curves have been analysed graphically with the help of the equation written by analogy with the equation derived to analyse the shape of the square-law response curve of the H_{01} resonator. The form in which σf, the off-resonance frequency shift and IDI the galvanometer deflection at $f_0 \pm \delta f$ frequency are fitted may be written:

$$1/IDI = 1/D_0\left(1+\tfrac{1}{2}\,\frac{\delta f^2}{f_0^{\,2}}\right)$$

Introduction

The E_{010} circular cylindrical cavity resonator was introduced by Jackson[1] to measure the dielectric properties of ideal solid dielectrics in the centimetre wavelength band. Later on the resonator was improved upon and modified to study both the solid and liquid dielectrics.

The mathematical analysis of the cavity resonator containing two and three dielectric layers and its internal electromagnetic field distributions were given in details by Jackson,[1] Dunsmuir and Poweles.[2]

The temperature variation of the resonant frequencies and the Q-curves of the air-filled and loaded cavity are studied. The Q-factor of the circular cylindrical cavity resonator is defined by:

$Q = W$[Stored Energy/Mean Power loss]$= f_0/df$. Where f_0 and df are the resonant frequency and half-power band width of the resonance curve respectively.

The shapes of the Q-curves of the air-filled and loaded resonator with square-law crystal detector may be graphically analysed with the help of the equation.

$$1/IDI = 1/D_0\left(1+\tfrac{1}{2}\,\frac{\delta f^2}{f_0^{\,2}}\right).$$

Where f_0 and D_0 are respectively the resonant frequency and galvanometer deflection i.e. resonance power output. The above equation is written by analogy with the equation derived by Collie, Hasted and Ritson[3] to analyse the shape of the response curve of the H_{01} resonator.

The E_{010} Cavity Resonator[4]

The E_{010} cavity resonator of radius $a = \lambda_0/2.6125 = 3.76$ cm and length, $l = 6$ cm (where λ_0 is the resonant wavelength of the air-filled cavity) is bored out of a solid brass cylinder with slightly larger radius than required and given a thick silver coating and then machined to obtain the required radius. The ends are fitted with two flat removable lids which are also coated with silver and secured by a set of screws. Care is taken during turning, reaming and lapping of the inner surface of the cavity so as to obtain constant diameter over the whole length (within ± 0.02 cm). Surfaces of the cavity and lids are kept clean and polished to obtain high quality factor. The E_{010} resonator is shown in Fig. 1.

Two 1-cm diameter holes are provided at diametrically opposite points on the curved surface of the cavity midway between the ends, to hold the input and output magnetic loop-probes. The semicircular loops, projected nearly 3 mm inside the cavity are made out of the inner conductor of Pyrotenax coaxial cable. The planes of the loops are kept parallel to the axis of the cavity in order to link with the magntic field. The loop-probes are fitted with screw and lock assembly so that the depth of penetration of the loops inside the resonator can be changed without altering the position of their planes to obtain best feed in and pick-up of power.

The specimen is held central along the axis of the cavity with the help of a 3.5-cm long brass plug with adjustable hole fitted on top lid and a hole drilled at the centre of the bottom lid (The specimen is cenrtal to within 0.1 mm at the two ends).

The E_{010} mode is the lowest frequency mode of the circular cylindrical cavity and is characterized (for perfectly conducting boundaries) by a purely longitudinal electric field and a purely circumferential magnetic field. The field distribution is shown in Fig. 2. In practical resonator the finite conductivity of the wall material and the presence of the probes produce deviations from the ideal field distribution but the overall

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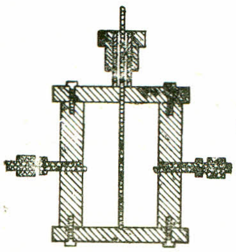

Fig. 1

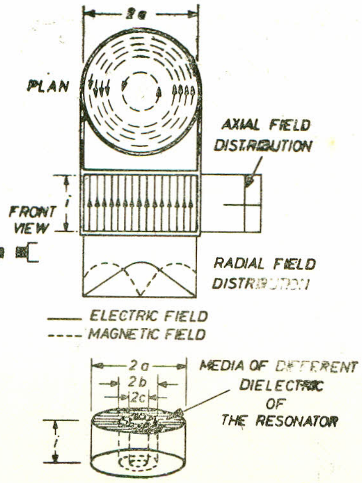

Fig. 2

Fig. 1.—The E_{010} circular cylindrical cavity resonator and the media of different dielectric.

Fig. 2.—The E_{010} resonator's field configurations.

effect is so small that for evaluation of resonant frequency, energy stored and power loss, the field distribution are assumed to be ideal. The presence of the specimen along the axis of the resonator does not destroy the circular symmetry of the electromagnetic field, but modifies the radial distribution.[5,6] The resonant frequency of the system then depends on the dimensions and dielectric properties of the specimen.

The length dimension of the cavity resonator does not enter into the resonance condition and its value is governed by the desire to obtain as high a theoretical Q-factor as possible, whilst avoiding the unwanted mode of oscillation. The calculated theoretical Q-factor of the cavity mode is nearly 20,000 according to $Q_T = al/d(a+l)$, where d is the depth of current penetration in the walls at the resonant frequency and the experimental Q-factor of the air-filled cavity is 5,000 i.e. 25% of the theoretical value.

Microwave Circuitry and Procedure of the Measurement

The circuitry for the study of the resonance of E_{010} cavity resonator in transmission is shown in the block diagram of Fig. 3. The microwave circuit components are: Klystron power supply, Reflex Klystron (Tunable 2.7–3.7 GHz), course and fine attenuators, three-screw-tuner, wave-guide to coaxial transformer, wavemeter (2.9 to 3.15 GHz), crystal detectors and galvanometers. The Pyrotenax Cable (Loss 10 dB/metre) and B.N.C (50Ω) plugs and sockets are used to make the coaxial lines and necessary connections.

After an initial warming up time of about an hour, the output power and frequency of the Klystron fed from a stabilized power pack is very stable, provided sufficient attention is introduced in the line with the help of attenuators. This also protects the Klystron from reflection and the consequent frequency pulling. A loss of several decibles of power in the microwave line must be allowed in order to avoid any possible interaction between the pieces of measuring equipment and the source, thus the source is isolated from the rest of the microwave line. The sensitive mirror-galvanometer with lamp and scale (Galvanometer sensitivity is approximately 0.01 μA/cm at 1m) is used to draw the resonance curves of air-filled and loaded cavity at different temperature.

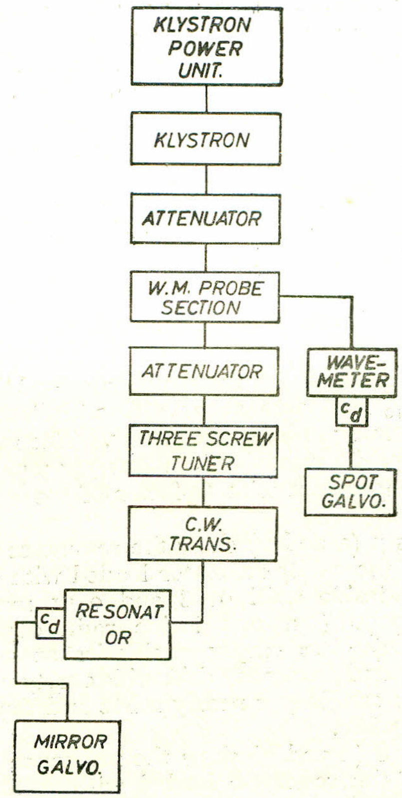

Fig. 3.—The block diagram of microwave circuitry.

The source frequency fed to the E_{010} cavity resonator is measured with a high Q-wavemeter (The accuracy of the frequency measurement is about ±0.025 Mc). The depths of penetration of the feed loops inside the resonator are adjusted to obtain maximum output power and the matching of the microwave circuitry is checked. It is done by varying the depths of penetration of the tuner screws inside the waveguide till the maximum output power is obtained. When the system is well-matched and the source is free from frequency pulling, maximum amount of power will be delivered to the load for the same source output power level. The crystal detectors are stored inside the expanded polystyrene in order to protect them from large temperature variation. The cavity resonator with feed cables are enclosed inside polythene bag (0.15 mm thick) and totally immersed into the liquid of thermostat bath (the accuracy of temperature control of the bath used is nearly ± 0.01°C in the temperature range from 0° to 120°C). Preheated dry air is passed slowly through the resonator before taking the measurement in order to eliminate any vapour (of bath liquid or water) that may enter inside the cavity which effect the resonator parameters.

Results and Discussion

Figure 4 shows the resonance curve of the air-filled E_{010} resonator at 21.5°C with increasing and decreasing order of frequency. The resonant frequency of the cavity is about 3045.5 Mc at above temperature with Q-factor of the order of 5,000.

Figures 5 (a and b) show the resonance curves drawn of the cavity resonator loaded with quartz (out and inside radii 0.68 and 0.31 mm) and corning 7070 (out and inside radii 0.58 and 0.373 mm) glass empty capillaries at 21.5°C. The measured frequency shift with empty corning 7070 glass capillary is nearly 2 Mc and that with quartz 3.5 Mc.

Figures 6 (a and b) show the resonance curves drawn of the resonator loaded with water-filled Corning capillaries at 20°C and quartz capillaries at 31.7°C. The Q-factor of the loaded resonator falls to the value of several hundred only.

In the present investigation small radii capillaries are used, with these the Q-curves are comparatively sharp and, therefore, the frequency shift can be measured accurately. The frequency spread of the Q-curves with water filled capillaries at the resonant deflection is well within 0.15 Mc. for frequency shift from 23 to 37 Mc.

It was thought profitable to write down a suitable equation, with the help of which, it may be possible to analyse the shape of the Q-curves obtained with the E_{010} cavity resonator and to check the square-law response of the crystal detector. The following equation is written to analyse the shape of the square-law crystal response curve of the E_{010} cavity resonator, by analogy with equation derived by Collie, Hasted and Ritson[3] for the shape of the response curve of the H_{01}-resonator.

$$|D| = f_0\, D_0/(f_0^2+\delta f^2)^{\frac{1}{2}} = D_0\{1+(\delta f/f_0)^2\}^{-\frac{1}{2}}$$

where f_0, D_0, δf and $|D|$ are respectively the resonant frequency, the galvanometer deflection at resonance, the off-resonance frequency shift and the galvanometer deflection at frequency $f_0 \pm \delta f$. Neglecting the higher terms of δf in the binominal expansion.

$$|D| = D_0\{1+\tfrac{1}{2}\,(\delta f/f_0)^2\}$$

$$1/|D| = 1/D_0\{1-\tfrac{1}{2}\,(\delta f/f_0)^2\}^{-1}$$

$$= 1/D_0\{1+\tfrac{1}{2}\,(\delta f/f_0)^2\}$$

$$= 1/2D_0 f_0\,(\delta f)^2 + 1/D_0$$

From the above equation a useful graphical tretament of the shape of Q-curve for the square-law crystal detector is to plot the experimentally ob-

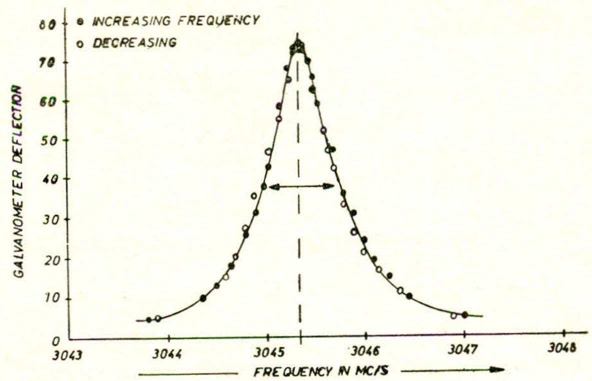

Fig. 4.—The resonance curve of the air-filled E_{010} resonator.

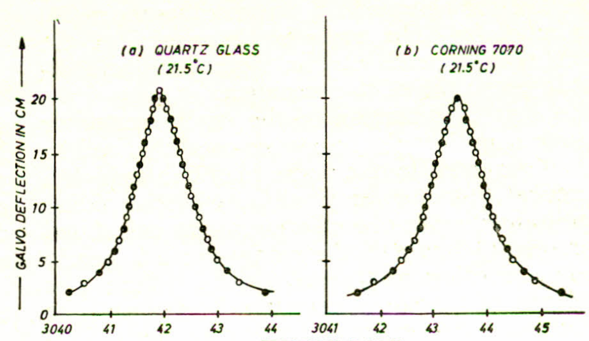

Fig. 5 (a and b).—The resonance curves of the E_{010} resonator loaded with empty capillaries of quartz and Corning 7070 glass.

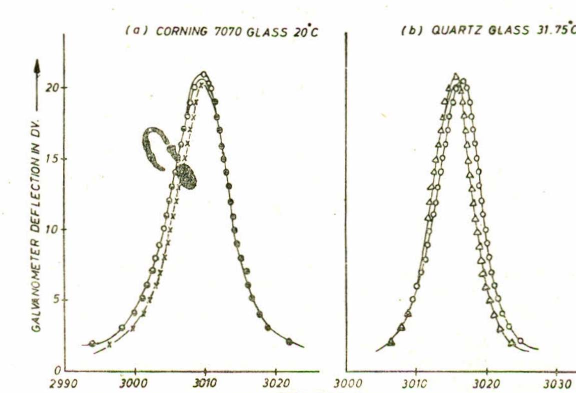

Fig. 6 (a and b).—The resonance curves of the E_{010} resonator loaded with water-filled Corning and quartz capillaries.

served δf^2 against reciprocal of the response $1/|D|$. The typical resonance curves of the air-filled and loaded resonator are shown in the Fig. 7 (a and b) together with the linear plots of analysed response curves i.e. δf^2 against $1/|D|$. Good straight lines are obtained. This justifies the use of the above equation for E_{010} resonator and provides a useful check on the square-law detection of the crystal detector.

The measurements of the resonant frequencies of the air-filled and loaded cavity resonator are made between the temperature 12 and 70°C with the temperature interval of about 5°C. The results are shown in the Tables 1 and 2 and are plotted in Figs. 8 and 9.

The plots of resonant frequency of the air-filled cavity resonator against increasing and decreasing order of temperature in the Fig. 8 show that the resonant frequency is decreased with the increase of cavity temperature. This may be explained as follows:

$a = \lambda_0/2.61$ and therefore $f_0 \propto 1/a$. That is to say, the resonant frequency is inversely proportional to the radius of the cavity resonator. Now a is

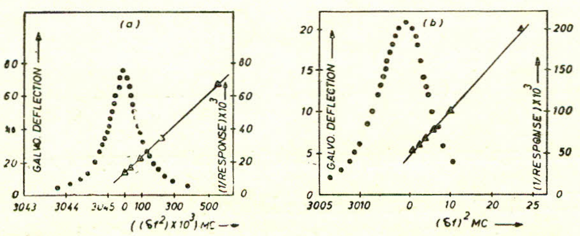

Fig. 7(a and b).—The typical resonance curves of the air-filled and loaded E_{010} resonator together with the linear plots of analysed response curves.

TABLE 1.—AIR-FILLED CAVITY RESONATOR.

Run 1 (heating)		Run 2 (cooling)	
Temp°C	Res. frequency (Mc)	Temp °C	Res. frequency (Mc)
12.25	3045.95	12.52	3046.10
17.25	3045.75	17.49	3045.70
22.26	3045.50	22.02	3045.6
27.25	3045.30	27.04	3045.4
32.73	3045.05	32.45	3045.1
37.75	3044.75	37.44	3044.8
42.26	3044.60	42.47	3044.7
47.25	3044.20	47.44	3044.3
52.25	3044.05	52.47	3044.15
57.25	3043.70	57.44	3043.8
62.75	3043.40	62.45	3043.60
67.25	3043.20	67.54	3043.20
70.26	3043.10	70.07	3042.90

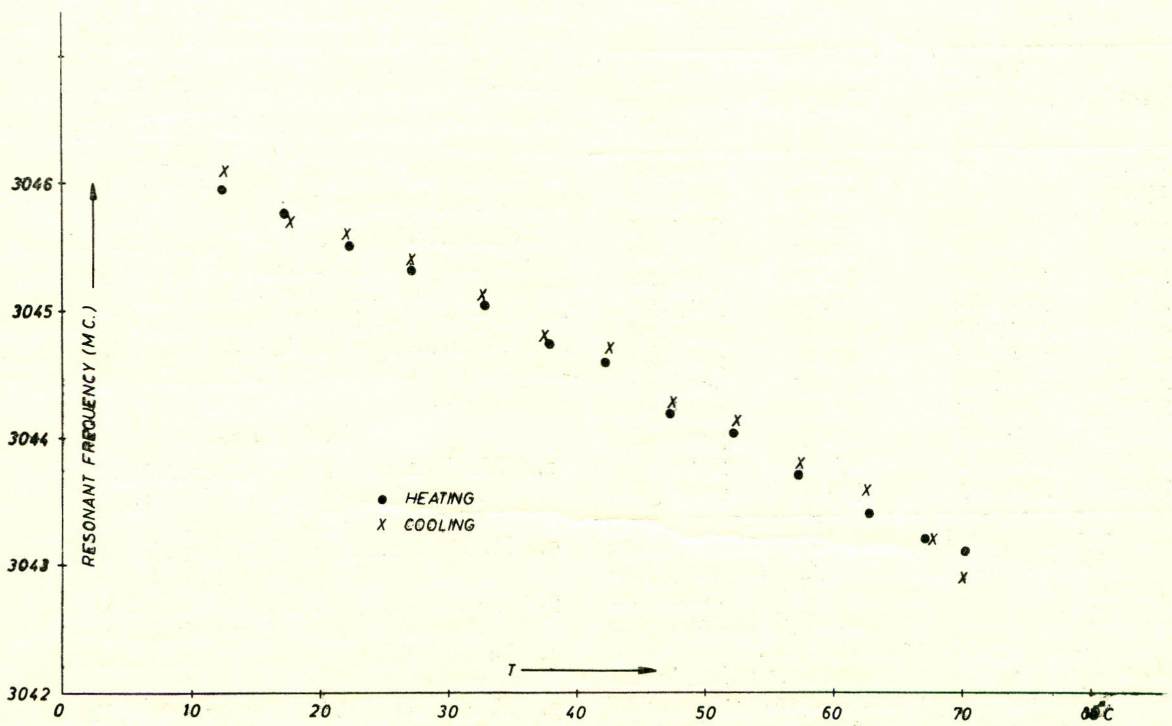

Fig. 8.—The plots of resonant frequency of the air-filled cavity resonator against increasing and decreasing order of temperature.

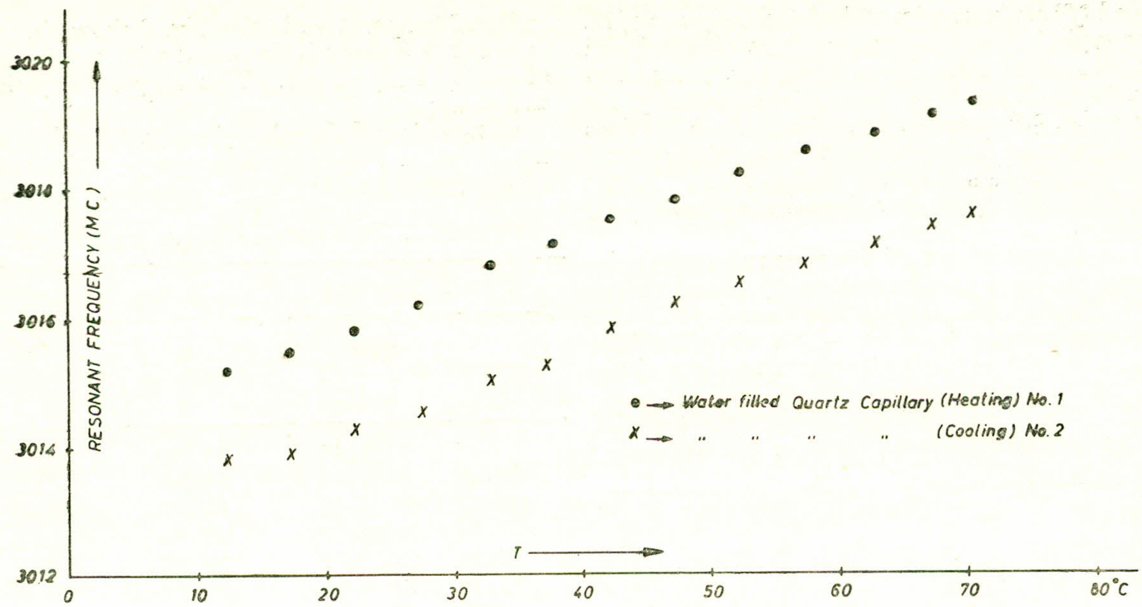

Fig. 9.—The plots of resonant frequency of the loaded E_{010} resonator against increasing and decreasing order of temperature.

TABLE 2.—LOADED CAVITY (WATER FILLED QUARTZ CAPILLARY).

Run 1 (heating) (Outside and inside radii 0.58 and 0.31 mm)		Run 2 (cooling) (Outside and inside radii 0.698 and 0.335 mm)	
Temp °C	Res. frequency (Mc)	Temp °C	Res. frequency (Mc)
12.25	3015.17	12.25	3013.78
17.25	3015.47	17.25	3013.90
22.26	3015.80	22.26	3014.25
27.25	3016.17	27.25	3014.60
32.73	3016.82	32.75	3015.00
37.75	3017.10	37.25	3015.25
42.26	3017.52	42.26	3015.78
47.25	3017.82	47.26	3016.22
52.25	3018.22	52.25	3016.52
57.25	3018.55	57.25	3016.78
62.75	3018.82	62.79	3017.08
67.25	3019.12	67.27	3017.38
70.26	3019.27	70.26	3017.58

directly proportional to the temperature of the cavity (i.e. $a \propto T$). Therefore, f_0 is inversely proportional to the temperature of the system i.e. $f_0 \propto 1/T$. The total variation of f_0 between the temperature 12–70°C is about 4 MC.

The plots of the experimental data points in the Fig. 9 with increasing and decreasing order of temperature for two different capillaries show the increase in f_{OL} against T. The total variation of f_{OL} between the temperature 12–70°C of the loaded cavity resonator is approximotely 3–4MC.

This may be explained as below:

Experimentally the value of ε' and ε'' (the real and the imaginary parts of the complex permittivity, $\varepsilon = \varepsilon' - J\varepsilon''$) are decreased with the increase of temperature [3,4] and also the resonant frequency of the specimen loaded cavity (f_{OL}) is inversely proportional to the permittivity of the specimen[2,4] ($f_{OL} \propto \frac{1}{\varepsilon}$; keeping all other pararmeters constant). From the above evidence it is seen that f_{OL} should be directly proportional to T ($f_{OL} \propto T$); and this is established by the experimental findings of the present investigation.

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