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FORMATION OF RESONANT STATES IN $K^-n \rightarrow \pi^\circ\Sigma^-$ BETWEEN 1850 AND 2160 MEV

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Results are presented for a partial-wave analysis of the $I=1$ reaction $K^-n \rightarrow \pi^\circ\Sigma^-$ covering a range of CMS energies from 1850 to 2160 MeV. The events used were obtained from the interactions of the type $Kd \rightarrow \pi^\circ\Sigma^-$ p_s. Values of the resonance parameters of the Y^*_1 (2030) were determined. In addition, it was found that $F_{5/2}$ amplitude resonates at $\sim$ 1980 MeV whose parameters differ significantly from Y^*_1 (1910) but in better agreement with the new Y^*_1 (1940) reported by Barnes *et al.*[5]

1. Introduction

Several Y^*_1 resonances have been reported to exist in the energy region 1800–2200 MeV. Of these, the Y^*_1 (2030) with spin parity $J^P=7/2^+$ is well established. Positive evidence for a Y^*_1 (1910) with $J^P=5/2^+$ has been found in total cross-section data [2,3] and in an analysis of several formation experiments.[4] In addition, a recent production experiment[5] suggests a resonance, Y^*_1 (1940) with parameters different from those of the Y^*_1 (1910). In particular there is a strong disagreement between the ratios of the $\pi\Lambda$ and $\pi\Sigma$ branching fractions for the two resonances.

In this paper we present results of an energy-dependent partial wave analysis of the pure $I=1$ reaction $K^-n \rightarrow \pi^\circ\Sigma^-$ covering the CMS energy region from 1850 to 2160 MeV.

2. Experimental Details

The experiment was performed at the Rutherford High Energy Laboratory by exposing the 80-cm. Saclay bubble chamber filled with liquid deuterium to a two-stage electrostatically separated K^- beam. Two exposures were made, at beam momenta of 1.45 and 1.65 Gev/c and yielding, respectively, 3.0 and 1.6 events per microbarn from a total of approximately 700,000 pictures. The motion of nucleons inside the deuteron enabled us to study K^-n interactions over a range of CMS energy from 1850 to 2160 MeV. The beam had a momentum resolution of $\pm 1\%$ found by kinematically fitting τ decays occuring inside the chamber.

The events under study include only those with one or two prongs with kink on one track. Long and curved kink tracks were rejected at the scanning stage in order to include only the strange particle decays. Events with an odd number of prongs or with a visible slow proton were selected as possible K^-n interactions and were measured on SMPS or conventional machines. They were then analysed using either the RHEL or CERN geometrical reconstruction and kinematical fitting programmes.

Events with invisible spectator protons were constrained in the kinematical fit to satisfy the following relationship between the 3 momentum components:

$$P_x = P_y = \tfrac{3}{4}\, P_z = (0.0 \pm 30) \text{ MeV/c}$$

the error allowing for the Hulthen distribution; the increased error in P_z takes into account the decreased detection efficiency for short proton tracks along the optical axis. The momentum distribution for inserted spectators joined smoothly onto that for seen spectators and agreed approximately with that predicted using a Hulthen wave function for the deuteron.

3. Selection of $K^-n \rightarrow \pi^\circ\Sigma^-$ Events

Since there is a missing π° involved in this reaction, therefore, the fits with unseen spectators should not be considered reliable for the partial wave analysis. In view of this events only with a seen spectator proton were retained for analysis. A total of 1589 events fitted this reaction channel out of which 545 with seen spectators <280 MeV/c. The following selection criteria were adopted to pick the best possible sample of events:

(i) *Missing Mass Selection.*—The reaction

$$K^-d \rightarrow \Sigma^- p\pi^\circ \quad (1)$$

could be ambiguous with

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$$K^-d \rightarrow \Sigma^- p \pi^\circ \pi^\circ \quad (2)$$

and

$$K^-d \rightarrow \Sigma^- p \eta^\circ \quad (3)$$

Figure 1 shows the spectrum of the square of the missing mass to the (Σ^-p) system. Cuts at masses of -0.2 and $+0.2$ Gev^2 separate an almost pure sample of $\pi^0\Sigma^-$ events.

(ii) *Weighting of the $K^-n \rightarrow \pi^\circ\Sigma^-$ Events.*—After selecting the $K^-n \rightarrow \pi^\circ\Sigma^-$ events on the basis of missing mass distribution, each event was weighed for (a) short decays, and (b) small angle decays.

Applying the above selection criteria we were left with 421 seen spectator events for final analysis.

4. Cross-sections for the Reaction $K^-n \rightarrow \pi^\circ\Sigma^-$

Average Cross-sections for Each Exposure.—Cross-sections were calculated separately for the two exposures using the complete Birmingham sample of films at 1.45 and 1.65 Gev/c. The following corrections were applied to calculate the cross-sections: (i) a factor of 1.17 to compensate for losses in scanning and processing, (ii) a factor of 1.05 ± 0.01 to correct for the Glauber shadowing of the neutron by the proton within the deuteron, (iii) weighting factor for short and small angle decays, and (iv) a factor of 1.13 ± 0.05 to allow for events excluded by the cut-off on spectator momentum at 280 MeV/c.

The cross-sections are presented in Table 1.

Variation of Cross-section With CMS Energy.—Assuming the validity of the impulse approximation the CMS energy E of the K$^-$n system for each event is equal to the effective mass of all secondaries except the spectator proton. To unfold the neutron cross-section, $\sigma(E)$, at a CMS energy E from theobse rved number of events, $N(E)\,dE$ in an interval dE centred on E we used the relation[6]

$$N(E)dE \propto E\,\sigma(E)dE \int \frac{B(p)d(p)}{p} \int \left| \phi_H \right|^2 F(p_s) p_s dp_s \quad (4)$$

where p is the beam momentum and $B(p)$ is the distribution of beam momenta, assumed to be Gaussian with a full width of 30 MeV for both exposures. ϕ_H is the Fourier transform of the Hulthen wave function for the deuteron, given by

$$\phi_H = \frac{(\beta^2 - \alpha^2)}{(p_s^2 + \alpha^2)\,(p_s^2 + \beta^2)}$$

with $\alpha = 45.7$ MeV; $\beta = 238$ MeV

and p_s is the laboratory momentum of the spectator proton. $F(p_s)$ is a flux factor taking account of the relative motion of the target and incident K$^-$ particle and is given by

$$F(p_s) = qE/pE_n \quad (6)$$

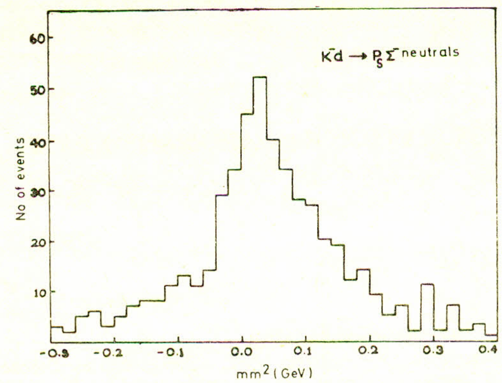

Fig. 1.—Missing mass squared distribution to the $p\Sigma^-$ system in the reaction $K^-d \rightarrow p\Sigma^-$—neutrals.

TABLE 1.—CROSS-SECTIONS.

Beam momentum (Gev/c)	Final state	Number of unique events	Corrected number	Cross section (mb)
1.45	τ	980	1060	1.48 ± 0.02
	$\Sigma^-\pi^\circ$	79	320	0.47 ± 0.06
1.65	τ	505	590	1.27 ± 0.02
	$\Sigma^-\pi^\circ$	116	213	0.46 ± 0.06

where q is the momentum of the incident kaon in the system where the neutron is at rest and E_n is the laboratory energy of the neutron. From the impulse approximation the momentum of the neutron is equal and opposite to that of the spectator proton and we put

$$E_n = E_s = (M_p^2 + p_s^2)^{\frac{1}{2}}$$

Therefore equation 6 can be written as

$$F(p_s) = \frac{qE}{p\,(M_p^2 + p_s^2)^{\frac{1}{2}}} \quad (7)$$

The energy variation of the cross-section as a function of K$^-$n CMS energy was obtained by comparing the observed distribution of fitted events with that predicted using the Hulthen wave function and the flux factor. In deriving this, only those events were used which had a spectator momentum between 100 and 280 meV/c. The cut-off at 100 MeV/c was chosen to avoid any possible bias due to scanning losses for seen spectators. The theoretical curves were obtained by integrating at a given E over the above range of spectator momenta. Figure 2 shows the experimental distribution and the predicted curve for events with $100 < p_s < 280$ MeV/c.

The whole energy range was divided up into bins and the cross-sections were calculated for each bin. The numerical values of the cross-sections and $Ao = \sigma/4\pi\lambda^2$ are given in Table 2 and plotted in Fig. 3.

5. Partial Wave Analysis

Basic Formulae.—At a given energy the differential cross-section for the reaction $K^- n \rightarrow \pi^0 \Sigma^-$ can be expressed as a series of Legendre polynomials

$$\frac{d\sigma(\theta)}{d\Omega} = \lambda^2 \sum_n A_n P_n(\theta) \qquad (8)$$

θ is the CMS scattering angle defined by $\cos\theta = \hat{K}.\hat{\pi}$ and A_n were expansion coefficients and P_n is the Legendre polynomial of order n.

The coefficients A^n are functions of the complex transition amplitudes $T_I\pm$ for states with $J = I \pm \frac{1}{2}$ and form a convenient common meeting ground for experiment and theory.

Experimental Determination of Legendre Expansion Coefficients.—For each centre of massenergy bin the quantities A_n/A_o were estimated by the method of moments according to the formula

$$\frac{A_n}{A_o} = (2n+1) \sum_i \frac{W_i P_n(\theta_i)}{\sum_i w_i} \qquad (9)$$

W_i is the Weight of an individual event. The errors in A_n/A_o are the usual standard errors of the weighted mean. Fixing the maximum value of n at 6 is consistent with assuming that only up to F waves take part in the interaction in our energy region. Figure 4 shows the data plotted as functions of energy. The curves show the fitted values from best fit described in section 6.

Parameterization of the Partial Wave Scattering Amplitudes.—The following parameterization has been chosen for the partial waves:

(i) Background Amplitudes: Background amplitudes were parameterized in terms of the real and imaginary parts of the amplitude. It was assumed to be constant, i.e.

$T_B = A + iC$

with A and C variable.

(ii) Resonant Amplitudes: Resonant amplitudes were described by the Breit–Wigner formula.

$$T_R = \frac{e^{i\phi}\,(x_e\, x_r)^{\frac{1}{2}}}{\left[\dfrac{2\,(E_R - E)}{\Gamma} - i\right]} \qquad (10)$$

E_R is the resonance energy and x_e and x_r equal Γ_e/Γ and Γ_r/Γ respectively where Γ_e and Γ_r are the partial widths in the elastic and reaction channels respectively and Γ is the total width, $\Gamma = \sum_i \Gamma_i$, the sum extending over all possible channels.

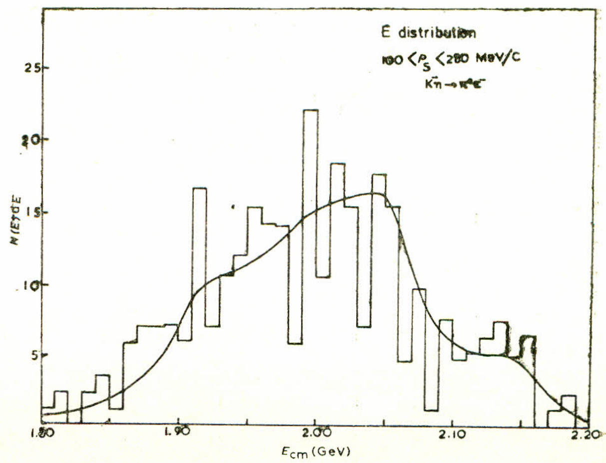

Fig. 2.—$N(E)\,dE$ vs E for $100 < p_s < 280$ MeV/c in the reaction $K^- n \rightarrow \pi^0 \Sigma^-$.

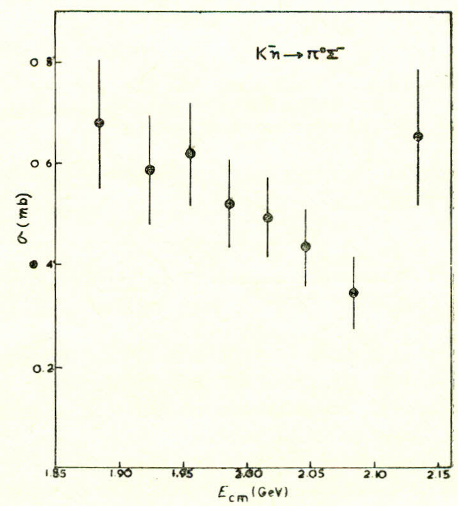

Fig. 3.— σ for the reaction $K^- n \rightarrow \pi^0 \Sigma^-$

TABLE 2.—BINS USED FOR THE DETERMINATION OF A_o (Seen Spectators $100 < p_s < 280$ MeV/c). The Reaction $K^- n \rightarrow \pi^0 \Sigma^-$.

Bin size (MeV)	Mean C.M. energy (M.V)	No. of weighted events	σ (mb)	$A_o = \frac{\sigma}{4\pi\lambda^2}$
1860–1910	1885.5	33.39	0.68±0.13	0.049±0.009
1910–1940	1924.0	34.52	0.59±0.11	0.047±0.009
1940–1970	1955.7	41.93	0.62±0.11	0.053±0.009
1970–2000	1986.9	42.22	0.52±0.09	0.048±0.008
2000–2030	2016.6	44.53	0.50±0.08	0.048±0.008
2030–2060	2046.7	40.39	0.44±0.08	0.045±0.008
2060–2110	2083.7	27.98	0.35±0.07	0.039±0.008
2110–2160	2135.1	30.54	0.66±0.14	0.081±0.017

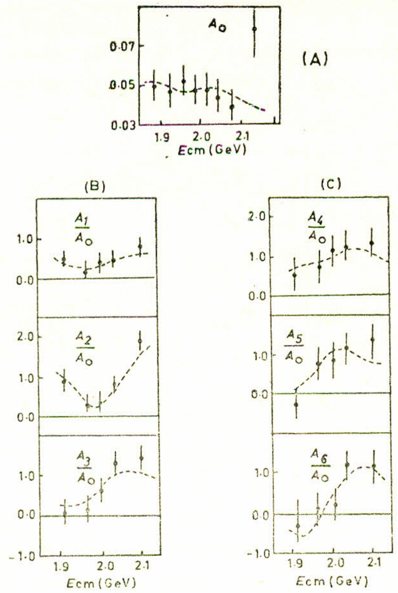

Fig. 4 (A, B, C).—Comparison of the A_0 and A_n/A_0 coefficients calculated from Fit 4 with those obtained from the experimental data for the reaction $K^- n \rightarrow \pi^0 \Sigma^-$

The partial widths are assumed to have the energy dependence[7]

$$\Gamma_i(E) \propto \left[\frac{p_i^2}{p_i^2 + x^2}\right]^{l_i} \left(\frac{p_i}{E}\right) \qquad (11)$$

Where x is a mass characterising the radius of interaction and p_i and l_i are the momentum and orbital angular momentum in the ith channel. ϕ is the phase at resonance.

Fitting Procedure.—Given a set of starting values for all the parameters used to describe the partial wave amplitudes, the amplitudes themselves and thence the quantities A_0 and A_n/A_0 could be calculated at each energy for comparison with their experimental values. Fitting was achieved by minimizing with respect to the parameters and the overall χ^2 given by

$$\chi^2 = \sum_{i=1} \sum_{j=1} (O_i - E_i) \; (G^{-1})_{ij} \, (O_j - E_j) \qquad (12)$$

Here O_i and E_i are the observed and expected values of the ith quantity and $(G^{-1})_{ij}$ is the inverse of the error matrix of the measured vasiables. The χ^2 function was then minimized with respect to all the parameters using the programme FMFP.[8]

6. Result of the Partial Wave Analysis

In Fit 1 we assumed all the amplitudes from $S\frac{1}{2}$ to $F_7/2$ to be constant backgrounds except $D_5/2$ which was described as a background + Breit–Wigner with the Y^*_1 (1765) fixed at its best value.[9] This simplest fit yielded a χ^2 of 48.1 for 24 degrees of freedom, a confidence level of 0.25% In the next fit (2) we replaced the $F_7/2$ background amplitude by a fixed Y^*_1 (2030) resonance; only the$(x_e\ x_r)^{\frac{1}{2}}$ was allowed to vary. The χ^2 went down to 23.4 for 25 degrees of freedom which is a significant improvement over fit 1. The value of $(x_e\ x_r)^{\frac{1}{2}}$ for Y^*1 (2030) obtained in this fit agreed well with the present world average.

According to the findings of Barnes *et al.*[5] a resonance of mass 1940 MeV and width 100 MeV was added to the $F_5/2$ amplitude. The $(x_e\ x_r)\frac{1}{2}$, E_R and Γ for this resonance were allowed to vary. The χ^2 came down to 15.2 for 22 degrees of freedom which corresponds to a confidence level of 85%. The $(x_e\ x_r)^{\frac{1}{2}}$ for Y^*_1 (2030) agreed quite well with the world average and the fitted values of mass, width and $(x_e\ x_r)\frac{1}{2}$ for the $F_5/2$ resonance were 1983 ± 25 MeV, 156 ± 80 MeV and 0.06 ± 0.02 respectively. Although the mass and width agreed tolerably with Y^*_1 (1940) within the assigned errors, the $\pi\Sigma$ branching fraction did not seem to be as large as mentioned by Barnes *et al.*[5] If we admit that there are two Y^*_1 S in the $F_5/2$ amplitude (Y^*_1 (1910) and Y^*_1 (1940) then our $F_5/2$ resonance is most probably the same Y^*_1 (1940), with mass and width a little bit different from those of ref. 5. In the next fit (fit 4), we held all the resonance parameters of Y^*_1 (2030) fixed at its best value and varied only the $F_5/2$ resonance. The solution was almost identical to fit 3 but it was a little improvement over fit 3 from χ^2 point of view. Because of the weak coupling of these resonances to the $\pi\Sigma$ channel and due to insufficient statistics it was not feasible to parameterize the phase of the resonance as an extra free parameter. However, we held them fixed either at 0° or 180° to achieve the best minimum. Fixing the phase of $F_5/2$ resonance at 180° (with $F_7/2$ resonance fixed at 180°) instead of fixing at at 0°, we obtained the same minimum but the amplitude of $F_5/2$ resonance turned negative relative to $F_7/2$ resonant amplitude, which proved that these two resonances were out of phase with one another. The summary of all the major fits are shown in Table 3 and the fitted parameters in Table 4. The amplitudes for Fit 4 are displayed in Fig. 5 and the fitted A_0 and An/Ao are shown in Figs. 3 and 4 together with the experimental data.

Summary of the Parameterization Used For Each Partial Wave Amplitude in the Reaction $K^- n \rightarrow \pi^0 \Sigma^-$.—$R$ stands for a Breit–Wigner resonant amplitude and C stand for nonresonant constant background amplitude. The resulting χ^2, number of degrees of freedom (n) and confidence level are given.

TABLE 3

Fit	Amplitudes							χ^2	n	C.L.
	$S_{1/2}$	$P_{1/2}$	$P_{3/2}$	$D_{3/2}$	$D_{5/2}$	$F_{5/2}$	$F_{7/2}$			
1	C	C	C	C	C+R	C	C	48.1	24	0.0025
2	C	C	C	C	C+R	C	R	23.4	25	0.5542
3	C	C	C	C	C+R	C+R	R	15.2	22	0.8535
4	C	C	C	C	C+R	C+R	R	15.2	23	0.8873

TABLE 4.—PARAMETERS AND QUANTUM NUMBERS OF Y* RESONANCES FOUND FROM THE ANALYSIS OF THE REACTION $K^-n \rightarrow \pi^0\Sigma^-$. THE QUANTITIES IN BRACKETS HAVE BEEN KEPT FIXED.

Fit	Mass E_R (MeV)	Width Γ (MeV)	Spin J	Parity p	$(x_{kn}x_{\Sigma\pi})^{\frac{1}{2}}$	Phase ϕ (Deg.)
	[1765]	[100]	5/2	—	[0.05]	[0]
3	1983±25	156±80	5/2	+	0.06±0.02	[0]
	[2030]	[120]	7/2	+	0.07±0.01	[180]
	[1765]	[100]	5/2	—	[0.05]	[0]
4	1985±21	159±80	5/2	+	0.06±0.02	[0]
	[2030]	[120]	7/2	+	[0.07]	[180]

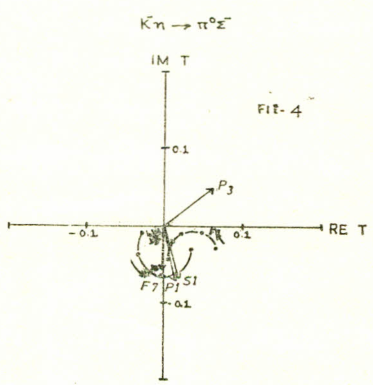

Fig. 5.—Argand diagrams for the partial waves (ReT vs. ImT) for Fit 4 in the reaction $K^-n \rightarrow \pi^0\Sigma^-$

7. Conclusions

From the analysis of the reaction $K^-n \rightarrow \pi\Sigma^-$, we have measured the branching ratio product $(x_{\bar{K}n}x_{\pi^0\Sigma^-})^{1/2}$ for Y^*_1 (2030) which agrees well with the world average. The resonance in $F_{5/2}$ amplitude appeared at a mass and width (E_R= 1985±21; Γ=159±80) which are entirely different from those of Y^*_1 (1910) but tolerably close to Y^*_1 (1940) within the assigned errors. The slight discrepancy may be due to insufficient statistics. We may infer that there could be two Y^*_1 resonances (Y^*_1(1910) and Y^*_1 (1940) in the $F_{5/2}$ amplitude with different decay frequencies for the $\pi\Lambda$ and $\pi\Sigma$ decay modes. The former has possibly no detectable $\pi\Lambda$ decay mode and conversely the latter has no detectable $\pi\Sigma$ decay mode. We should admit that this mystery of the $F_{5/2}$ amplitude still remains. This can be resolved unambiguously if a sufficient amount of data is available at and around 1900 MeV region.

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