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THE NEUTRINO HELICITY

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Introduction

The neutrino was conceived by Pauli–Fermi as a particle with vanishingly small mass, neutral charge, spin $\frac{1}{2}$ and following Fermi–Dirac statistics, and proposed as a remedial measure to remove certain anamolies in beta emission. Its mass was deemed to be $< \frac{1}{2000} \times$ mass of electron.

The author had earlier postulated the neutrino to be of an isotopic spin invariant of the electron, firstly on classical considerations and secondly on quantum mechanical considerations. He had pointed out the important roles the neutrino plays in beta emission,[1] the breakdown of the mass energy conservation laws in very strong fields,[2,3] violation of parity laws in weak energy interactions, multiplicity of elementary particle[4,5] and the weak forces.[6] One aspect of the neutrino, not dealt with by the author in previous papers, was the helicity of the neutrino. The helicity of the neutrino has been experimentally determined to be of the left spin type, as also predicted by theory. No right-handed neutrinos have been discovered so far. Could these exist? If so, why have these not been discovered? If, at all, how could the right-handed neutrino be discovered? The main purpose of this paper is to analyse and come to reasoned conclusions on these aspects.

The Alternative Modes of Pion Decay

The author may take this opportunity of mentioning about the μ-neutrino, having a half life of $\sim 2.1 \times 10^{-6}$ sec emerging out of the alternative modes of pion decay. The pion decays in two alternative modes viz. $\pi \rightarrow e + \nu$ or $\pi \rightarrow \mu + \nu$ in the first mode the μ-neutrino is emitted alongwith electron. For a pseudovector coupling, the electron mode is suppressed enormously and can be calculated theoretically:[7]

$$\text{ratio } \frac{\pi \rightarrow e + \nu}{\pi \rightarrow \mu + \nu} = \frac{\rho_e}{\rho_\mu} \frac{1 - v_e}{1 - v_\mu}$$

$$= \frac{m_e^2 (m\pi^2 - m_e^2)^2}{m_\mu^2 (m\pi^2 - m^2\mu)^2} = 1.3 \times 10^{-4}$$

After some failures, the electron decay of pion has been observed with counters and bubble chambers and finally it was quantitatively studied with a magnetic spectrometer. The results of the branching ratio obtained are in conformity with the theory. Experiments performed[8] on 33 BeV accelerator at Brookhaven, have established tracks of muons which were observed as expected from the interaction of μ-like neutrinos with matter.

The electron–muon mass difference is very substantial. If the muon is a Dirac particle with no anomaly, its gyromagnetic ratio, g, including quantum electrodynamical corrections is expected to be:

$$g = 2\left(1 + \frac{\alpha}{2\pi} + \frac{3\alpha^2}{4\pi^2} + \ldots\ldots\right) = 2 \times 1.00116$$

In an experiment at Cern,[9] the muons were stored in a magnetic field for more than 1500 turns. From the differential of longitudinal polarisation at the entrance and exit, one obtains the value of g in accordance with the theory. No significant difference between muon and electron is revealed, and the muon is a Dirac particle. Muon is accordingly like an electron except for the rest mass and the half-life decay period. The same analogy must run for neutrino and μ-neutrino.

Obviously, the muon in muonium (a neutral system consisting of muon and electron on the pattern of positronium), can decay into an electron and two neutrinos with a characteristic mean life. But the question arises whether muonium can annihilate into two photons (γ rays) as positronium does. It has been found experimentally that the two quantum annihilation of electrons and muons does not occur. The absence of muon annihilation, despite the fact that muons and electrons separately behave as simple Dirac particles, furnishes further evidence for the basic difference between muons and electrons. Of course, this could be explained by the absence of interference of two De Broglie waves. The differences therefore, are exactly similar to those between μ-neutrino and neutrino, in their rest mass difference, their difference in reacting with matter (the first gives muons and the second electrons) and in their stability (half-life decay period). Thus it can be said with a degree of certainty that these concepts have not changed our approach to problems on the basis of the neutrino, as postulated by the author.

The Theory of Weak Interactions with Parity Violation

The theory of weak interactions with parity violation may now be recapitulated. We will make use of relativistic pseudoscalar operator ν_5 to conform with usual signs and nomenclature of the equivalent Hermitian operator.

$$\Gamma_5 = -\iota\nu_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Introducing the parity mixing operators

$$\Lambda_\gamma = \tfrac{1}{2}(1+\Gamma_5) \quad \Lambda_\iota = \tfrac{1}{2}(1-\Gamma_5)$$

which satisfy the relations appropriate to projection operators

$$\Lambda_\gamma + \Lambda_\iota = 1 \qquad \Lambda_\iota\Lambda_\gamma = \Lambda_\gamma\Lambda_\iota = 0$$

$$\Lambda_\gamma\Lambda_\gamma = \Lambda_\gamma \qquad \Lambda_\iota\Lambda_\iota = \Lambda_\iota$$

and $\Lambda_\gamma{}^+ = \Lambda_\gamma \qquad \Lambda_\iota{}^+ = \Lambda \quad \Lambda_\iota\Lambda_\mu = \nu_\mu\Lambda_\iota$

$$\Lambda_\iota\,\nu_\mu = \nu_\mu\Lambda_\iota$$

These are the right and left helicity projection operators since Λ_γ and Λ_ι select right- and left-handed neutrions respectively.

Writing Λ_ι in matrix form

$$\Lambda_\iota = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

When it is applied to a Direc spinor in a positive energy state:

$$\Lambda_\iota\,\varphi = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \xi \\ \dfrac{\sigma.p}{/E/+m}\xi \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \xi - \dfrac{\sigma.p}{/E/+m}\xi \\ -\xi + \dfrac{\sigma.p}{/E/+m}\xi \end{pmatrix}$$

On the old theory of the neutrino $m=0$. Hence

$$\Lambda_\iota\varphi_\nu = \frac{1}{2}\begin{pmatrix} \xi_\nu - \dfrac{\sigma.q}{q}\xi_\nu \\ -\xi + \dfrac{\sigma.q}{q}\xi_\nu \end{pmatrix} = \begin{pmatrix} \xi_\nu \\ -\xi_\nu \end{pmatrix} \text{ if } \frac{\sigma.q}{q} = -1$$

(left helicity)

$$0 \text{ if } \frac{\sigma.q}{q} = +1 \ldots (y)$$

(right helicity)

When applied to electrons or equal massed neutrinos whose masses do not vanish, the helicity operators select partially polarised beams. The relative probabilities P_γ and P_ι for right and left-handed polarization ($\sigma.p = \pm p$) in the state $\Lambda_\iota\varphi_e$

$$P_\gamma = \left[1 - \frac{p}{/E/+m}\right]^2$$

$$P_\iota = \left|1 + \frac{p}{/E/+m}\right|^2$$

and thus the left-handed polarisation of the selected electron/neutrinos beam is

$$-P = \frac{P_\iota - P_\gamma}{P_\nu + P_\iota} = \frac{4\,[\,p\,/\,(/E/+m)]^2}{2 + 2p/(/E/+m)^2}$$

$$= \frac{p(/E/+m)}{E^2 + m\,/E/} = \frac{p}{/E/} = v$$

Agreement is obtained with observed helicities if the lepton wave functions φ in the weak interaction Hamiltonian are replaced by their left-handed components $\Lambda_\iota\Psi_\nu$

When the new matrix element is squared and averaged by the method of traces, it is sufficient to write the left-hand projection operator only once. Thus the new transition-probability consists of separate parity-conserving and parity-violating parts. It could be seen that g^2 in the old theory is replaced by $(\sqrt{2}\,g)^2.\frac{1}{2} = g^2$ in the parity-conserving part and by $-(\sqrt{2}\,g)^2\frac{1}{2}\Gamma_5 = -g^2\Gamma_5$ in the parity-violating part of the new transition probability.

Introducing a special symbol for the left-handed components $\Delta_\iota\varphi \equiv \chi$ the leptic part of the matrix element for β decay must be written

$\bar{\chi}\,\nu_\mu\,\chi_\nu$ for vector coupling

$\bar{\chi}\nu_5\nu_\mu\chi_\nu$ for axial vecter coupling

The nuclear factor must contain only vector and axial terms with

Vector coupling constant $\sqrt{2}\,g$

Axial coupling constant $\sqrt{2}\,Rg$ $(/R/ \simeq 1.15)$

$$M = \sqrt{2}\,g[\bar{\Psi}\Psi(1,\ldots,A\sum_\chi (T^+\nu_\mu)_x\Psi(1,\ldots,A)]$$
$$(\bar{\chi}_e\nu_\mu\chi_\nu) - \sqrt{R}\,Rg\,[\bar{\Psi}_{\Psi'}(1,\ldots,A)$$
$$(\sum_x(\tau^+\Gamma_5\nu_\mu)\Psi_1(1,\ldots,A)](\bar{\chi}_e\Gamma_5\nu_\mu\chi_\nu) + h.c.$$

But the leptic factors turn out to be identical

$$\bar{\chi}_e\nu_\mu\chi_\nu = \bar{\chi}_e\Gamma_5\nu_\mu\chi_\nu$$

M can be written as

$$M = \sqrt{2}\;g\,[\bar{\Psi}_F(1,\ldots,A)\Sigma_x[T^+(1-R\Gamma_5)\nu_\mu]_x.$$
$$\Psi_1[(1,\ldots,A)\,(\bar{\chi}_e\nu_\mu\chi_\nu)] + h.c.$$

Non-relativically, in the nucleon this becomes:

$$M = \sqrt{2}\,g\left(\int 1\right)(\bar{\chi}_e\nu_4\chi_\nu) + \sqrt{2}Rg\left(\int \sigma\right).$$
$$(\bar{\chi}\nu_4\sigma\chi_\nu) + \text{h.c.}$$

$$= \sqrt{2}\,g\left(\int 1\right)(\bar{\chi}_e\nu_4\chi_\nu) - \sqrt{2}Rg\left(\int \sigma_+\right).$$

$$(\bar{\chi}_e\nu_4\,\sigma_-\,\chi_\nu - \sqrt{2}\,Rg\left(\int \sigma_-\right).$$

$$(\bar{\chi}_e\nu_4\sigma_+ + \chi_\nu) + \sqrt{2}\,Rg\left(\int \sigma_z\right)\left(\bar{\chi}_e\gamma_4\sigma_z\,\chi_\nu\right) + \text{h.c.}$$

For allowed transition $\Psi_{\nu,e}$ is replaced by $\omega_{\nu,e}$ and $\chi_{\nu,e}$ by $\frac{1}{2}(1-\Gamma_5)\,\omega_{\nu,e}$.

We must calculate the average square of the matrix element for polarised nuclei and insert a projection operator $\Lambda_t=\frac{1}{2}(1-\Gamma_5)$ to select left-handed neutrinos which will automatically select left-handed electrons also.

When the new matrix element is squared and averaged by the method of traces, it is sufficient to write the left-hand projection operator only once. The new transition probability consists of separate parity-conserving and parity-violating parts. For the parity-violating part the calculation of the trace must be repeated after inserting a factor $-\Gamma_5$ to the left of the neutrino projection operator $/c/$.

The contribution of parity violation vanish for the non-relativistic vector interaction (γ_4) for the axial vector terms without spin flip $(\gamma_4\sigma_z$ or $-\Gamma_5\gamma_3)$:

$$Tr[\gamma_4(-\Gamma_5)/c/\nu_4(/c/+m)]=0$$
$$Tr[\Gamma_5\gamma_3(-\Gamma_5)/c/\Gamma_5\gamma_3(/c/+m)]=0$$

However, for spin flip $\nu_4\sigma\pm=-\Gamma_5\gamma\pm$ the parity violating trace does not vanish and its contribution describes the asymmetries.

We obtain $\frac{1}{4Eq}Tr[\Gamma_5\gamma\pm(-\Gamma_5)/c/$
$\gamma\pm\Gamma_5(/c/+m)]=\mp v\cos\theta_{eJ}\pm\cos\theta_{\nu J}$.

For a nuclear transition with $\Delta M=-1$ (nuclear term in $\int\sigma_-$, leptic term in σ_+) we must use the upper signs.

We obtain that the electrons of Co^{60} go predominantly backward with asymmetry coefficient v and that the neutrinos go predominantly forward with asymmetry coefficient 1, which is in accord with experiments. The formulas developed are for β^- emission. The results for β^+ can be similarly derived and are also in agreement with the experiment.

Five Possible Theories of β-decay

Having recapitulated the parity conservation and the parity violation theories and the left helicity operator functioning in weak interactions, as also the predominantly e—ν asymmetry effect, let us work out the five different theories of beta decay from Dirac spinors and show that the electron–neutrino correlation worked out shows clear evidence of the existence of left-helicity neutrinos and also right-helicity neutrinos, in consonance with their already khown electron counterpart and there being no theoretical concept or evidence to the nonexistence of the right-helicity neutrino.

As mentioned above, based on the five tensors that may be constructed from Dirac spinors, we may formulate five different theories of β decay consistent with the principles of conservation and with relativistic invariance.

Let us introduce the five tensor operators $Q_x(x=S,V,T,A,P)$ for scalar, vector, tensor, axial-vector, pseudoscalar. Then the matrix element for any one of these theories can be written:

$$M_x=g_x[\bar{\Psi}_{\Psi'}(1,\ldots,A)\sum_x(\tau^+Q_x)\Psi_1(1,\ldots,A)].$$
$$(\bar{\Psi}Q_x\Psi_\nu)+\text{h.c.}$$

It is seen that Fermi's theory is the particular case x=v. In the allowed approximation (non-relativistic nucleus pR and $qR<<1$) the matrix element becomes

$$M_S=g_S\left(\int 1\right)(\bar{\omega}_e\, 1\,\omega_\nu)+\text{h.c.}$$
$$M_V=g_V\left(\int 1\right)(\bar{\omega}_e\,\nu_4\omega_\nu)+\text{h.c,}$$
$$M_T=g_T\left(\int\sigma\right)\cdot(\bar{\omega}_e\,\sigma\cdot\omega_\nu)+\text{h.c.}$$
$$M_A=g_A\left(\int\sigma\right)\cdot(\bar{\omega}_e\nu_4\sigma\omega_\nu)+\text{h.c.}$$
$$M_P=g_P[\Psi_F{}^+(1,\ldots,A)\sum_x(\tau^+\sigma)\,x\cdot\Psi_1(1,\ldots,A)]\,d^3x_1\ldots d^3x_A$$

S and V give Fermi selection rules (zero pole); T and A give Gamow–Teller selection rules (magnetic dipole); P gives zero in the static case, since the matrix element contains the velocity $\nu_4\nu_1$ and would lead to a selection rule $\Delta J=0$, $\pi_F\neq\pi_1$, in contradiction with experiment. Thus we can discard this form of the theory at least for the discussion of allowed β decay,.

The leptic factors can be computed and summed over spins. The following results are obtained which describe the electron neutrino angular corelations predicted by different theories:

$$\frac{m}{E}\,\frac{m_\nu}{q}\int_e\int_\nu|\text{leptic factor}|^2$$
$$=1+\lambda_x\,v\cos\theta_{e\nu}\ \text{with}$$
$$\lambda_S=-1$$
$$\lambda_V=+1$$
$$\lambda_T=+\tfrac{1}{3}$$
$$\lambda_A=-\tfrac{1}{3}$$

In the spin coupled Gamow–Teller interactions, we take the axial-vector case and compute for $\Delta_{mj}=\pm1$,

We obtain

$$\frac{m}{E}\,\frac{m_\nu}{q}\int_e\int_\nu|\bar{\omega}_e(i\gamma_5 y\pm)\omega_\nu|^2$$
$$=1-v\cos\theta_{eJ}\cdot\cos\theta_{\nu J}$$

the average result agrees with the above.

The left spin of e± at weak energy levels and its reversal and pairing can be derived directly from Dirac's equations. The same pattern would be followed by ν, which is the isospin invariant of $e\pm$.

The Right Helicity Neutrinos

Thus the theoretical possibility for left- and right-helicity neutrinos clearly manifests itself, as well as the left-helicity neutrino already known in the laboratory. From the above equations and from equations Y and the laws of symmetry, one must, as well, look forward to the existence of right-helicity neutrinos. The reason for their not being detected so far is explained by the foregoing theory of weak interactions itself i.e. the neutrino should have left helicity. Once we come to the region of strong interactions, we can expect to observe right helicity (or right spin) neutrinos. Unfortunately the most powerful synchroproton that has provided us (Brookhaven 33 BeV) neutrinos, which only touch the threshhold of weak interaction in as much as the neutrino is involved. This is presumably why we have only observed left-spin neutrinos so far.

If a move is made to have more energetic beam of neutrinos, from more powerful accelerators under way, or from powerful enough linear accelerators in advanced countries, it could be predicted that we shall come across right helicity neutrinos.

Another possibility to observe them will be in deep space or on the moon where strong beams of neutrinos strike directly from the sun.

The helicity of neutrinos can be measured by means of a scatterer and transmission polarimeter and measuring the circular polarisation of γ-rays. If the γ-rays have negative helicity, they correspond to negative helicity neutrinos. It should not be difficult to preadjust the response of the polarimeter to only right helicity gamma rays—corresponding to right-spin neutrinos—and in the event of being so, actuating a signalling device. If such a compact apparatus is developed with a proper filter (lead) to eliminate other particles, an appropriate scatterer and a preadjusted polarimeter to respond to only right neutrinos bringing into action a signalling device, this apparatus or small gadget when exposed to radiation from the sun on the moon will at once send signals to earth confirming the existence of right helicity neutrinos.

Simultaneously, the hunt for right-helicity neutrinos may be continued in powerful linear accelerators, which could be a source of powerful beam of neutrinos, where they shake off their left helicity stamp of weak interactions. Technique and technology having developed beyond expectations, the author expects the confirmation of existence of right-helicity neutrinos in the very near future.

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