THE CONCEPT OF MATTER WAVES, ELECTROMAGNETIC WAVES AND SCATTERING

Part II.—Some Deductions from the New Postulates and Extension of the Theory

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The consequences of the two new postulates introduced in Part I of this series are developed. Since the momentum of the associated waves $h\nu/v$ equals $m_0 v/\sqrt{1-v^2/c^2}$, the momentum of the particle, it is seen that the momentum of the particle and its vector character are wholly accounted for by the momentum of the associated waves and the unconverted mass of the particle retains its scalar character and is tied up with its associated waves with the help of a binding energy.

Assuming that the two parts of associated waves derived respectively from the external work and the parent mass retain their separate identities, two separate names 'kinetic energy waves' and 'matter waves' are given to these parts respectively. The various phenomena of emission and absorption of radiation are then logically explained on the basis that radiation is always absorbed in the 'matter wave' form only and that in any single transition, radiation is emitted either in the form of 'kinetic energy waves' entirely or in the form of 'matter waves' entirely. It is shown that radiation in transit consists of waves of both the types. It is then postulated that the 'matter waves' in transit (indentified as photons) adjust themselves so as to become coherent with any closely accompanying 'kinetic energy waves' (identified as electromagnetic waves) of the same frequency.

The law of conservation of linear momentum is applied to fast electrons going from one field to another and also to reflection and refraction of 'kinetic energy waves' and 'matter waves' or photons assumed to be moving with different velocities in different media. It is shown that the electrons follow the law $\sin \theta_i/\sin \theta_r = (v/u)\sqrt{1-u^2/c^2}/\sqrt{1-v^2/c^2}$ which can be tested experimentally, and the waves and photons follow the usual laws of reflection and refraction applicable to light waves.

Finally it is stated that from the results of the experiments on excitation of atoms and emission of X-rays with the impact of high velocity electrons we can calculate the energy used in binding the waves of the two types corresponding to different velocities of the electron.

1. Introduction

In Part I of this paper, it was shown that certain parts of de Broglie's original postulate were exposed to several objections, and it was tentatively suggested that those parts should be dropped and instead the fundamental de Broglie postulate $\lambda = \frac{h}{mv}$ should be combined with two new ones namely that the velocity of the associated waves is *equal* to the velocity of the *particle* and that their energy and frequency are given by

$$E^* = h\nu^* = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} \qquad (1)$$

It was shown that the new postulates lead to the new result that out of the total energy, $\frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$, of the moving particle, a part, $\frac{m_0 v^2}{\sqrt{1-v^2/c^2}}$, is carried by the associated waves, and the remaining part $m_0 c^2 \sqrt{1-v^2/c^2}$ is localised in the particle.

In the present communication, it is proposed to extend the theory, to elaborate some consequences of the new postulates, and to show how these postulates lead to a more satisfactory alternative explanation of some of the phenomena of atomic physics.

2. Momentum

From the fact that radiation exerts pressure, it can be shown in an elementary way that the momentum of a photon of frequency ν is given by $h\nu/c$. In a similar way, it can be shown that the momentum of the waves of energy $h\nu^* = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}}$ associated with a particle of rest mass m_0 and moving with a velocity v is given by

$$\frac{h\nu^*}{v} = \frac{m_0 v}{\sqrt{1-v^2/c^2}}, \qquad (2)$$

which is known from the theory of relativity[1] to be the momentum of the particle itself. This immediately leads us to an important conclusion:

Momentum of a moving particle is entirely accounted for by the momentum of its associated waves; so that the vector character of the moving particle is entirely due to the waves associated with it, while that part of it (the particle) in which the energy not in the wave form (equal to $m_0 c^2 \sqrt{1-v^2/c^2}$) is localised, continues to have

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its scalar character. It must, however, be tied up with its associated waves with the help of some binding energy, so that it is forced to follow the path along which its associated waves guide it.

3. Classification of Associated Waves

We can already notice from Part I of this paper (equations 5 and 6) that of the total amount of wave energy (equal to $\frac{m_o v^2}{\sqrt{1-v^2/c^2}}$), only a part equal to $\frac{m_o c^2}{\sqrt{1-v^2/c^2}} - m_o c^2$ is derived from the external work done on the particle for increasing its velocity, while the remaining part is derived from the conversion of matter into wave energy. We may tentatively assume that the two parts of wave energy derived from two independent sources retain their separate identities (but are bound together with the help of a binding energy), and the two together are bound to the particle with the help of the binding energy mentioned at the end of section 2 above.

We shall denote the first-mentioned part of the waves derived from external work as kinetic energy waves, and the other part derived from conversion of matter into energy as matter waves.

4. Emission and Absorption of Radiation

When an unexcited electron with total energy equal to W_1 absorbs an amount of energy equal to $h\nu$ and jumps over to a higher orbit with total energy equal to W_2, its potential energy in the higher orbit is higher by $2h\nu$ and its wave energy lower by $h\nu$, so that its total energy has increased by $h\nu$, while the energy in the wave form has decreased. It is reasonable to assume that emission of radiation does not take place before the electron starts jumping back to its unexcited state, because the final orbit in which the electron is going to land is not known. It will also not be correct to assume that the phenomenon of radiation takes place continuously as the electron is in transit towards the final lower orbit because (in view of the fact that radiation consists of *finite* and *continuous* wave trains) the speed of the transit of electron in between the orbits will have to correspond to the speed with which emitted radiation is propagated, which does not seem feasible. We, therefore, conclude that the phenomenon of radiation takes place when the electron has arrived at an orbit where it can be stably accommodated. As the electron leaves the orbit in which it is in an excited state, the potential energy released is wholly converted into kinetic energy and the speed of the electron keeps on increasing till it arrives at the final lower orbit, with knietic energy increased by $2h\nu$. The energy in the matter wave form has increased by a corresponding amount, potential energy has decreased by $2h\nu$; the total energy being the same as before is more than the amount required for that lower orbit by $h\nu$, and the electron is moving too fast for that orbit. The unwanted excess energy equal to $h\nu$ is in the wave form and is instantly radiated out. The emitted radiation carries its momentum with it and the momentum and the speed of the electron are thereby reduced to the values appropriate for that orbit. In view of the fact that the two forms of waves are joined together by means of some binding energy we are led to the following postulate:

Postulate III.—In any single transition, radiation can be emitted either in the form of kinetic energy waves or in the form of matter waves, but never partly in one form and partly in the other. It is always absorbed in the form of matter waves, which are identical with the commonly known photons.

Thus, the energy radiated out is taken from the wave energy either wholly in the matter wave form or wholly in the kinetic-energy wave form. Calculations show that, in general, sufficient amount of energy for radiation is available in both the forms. We shall call the waves emitted in any single transition, whether of one type or of the other, as a wave packet. Since momentum is proportional to the corresponding wave energy, irrespective of its form, kinetic energy wave or matter wave, the conditions of equilibrium of this electron in its orbit are the same whether the energy radiated belongs to one form or to the other. If once again this electron jumps to the same higher orbit as before and then goes back to its initial orbit, the radiation emitted by it may or may not be of the same form of wave energy. If after $m+n$ emissions of radiation of the same frequency by this electron it is in its unexcited state and is also in its original condition as regards the distribution of energy between the two types of waves, while during the intervening period it has emitted kinetic energy waves m times and matter waves n times, then in the radiation of the corresponding frequency emitted by the atoms of the substance in question, the energy distribution between the two types will be in the ratio $m:n$. Calculation shows that the rate at which energy in the matter wave form is supplied in the transitions under discussion is always less than, but more than one half of (and in most cases nearly equal to) the rate at which energy in the kinetic energy wave form is supplied. It follows that m is always greater than n, less than $2n$, and in most cases nearly equal to n.

In the foregoing discussion it has been assumed for simplicity that the transitions are all of the same kind between the same two orbits. But the transitions could, without affecting our argument, occur between the given orbit and any of the higher orbits. The points to be made out are (a) that each electron does return to the unexcited state with the normal distribution of energy between the two types of waves after a finite number of transitions, and (b) in the emitted radiation of any given frequency, the energy emitted in the kinetic energy wave form is always more than that in the matter wave form.

Before this radiation leaves the orbit, it is moving with the velocity of the associated waves, which is equal to the velocity of the electron with which it was associated. As soon as it leaves the atom, it becomes radiation in transit, and the first change it undergoes (instantly) is that its velocity is now equal to the velocity of light, which is much greater than the velocity of the electron. If the velocity is u immediately before radiation and c after it has been radiated out, then each wave of original length λ will after radiation have its wave length elongated to $\lambda' = (c/u)\,\lambda$, which is in accord with the relation $\nu = \frac{u}{\lambda} = \frac{c}{\lambda'}$. With the increase of velocity its momentum decreases, and so in pulling itself out of the electron with which it was associated, the radiation loses a part of its momentum which goes not to the associated waves but to the atom as a whole.

5. Reflection

Waves as well as photons carry momentum, and their path after reflection whether determined by the application of the principle of conservation of linear momentum along and perpendicular to the reflection surface or obtained by Huygen's construction for waves is the same.

6. Refraction

(*a*) *Refraction of High Speed Electrons.*—Let us consider once again the pencil of electrons considered in subsection IV of section 2 of Part I of this paper. If the incident pencil proceeds along AB (Fig. 1 of Part I) with a velocity u and the refracted one along CD with a velocity v, then since there is no force acting on the electrons perpendicular to the direction of the field, the resolved part of the momentum of each electron perpendicular to the field must remain unchanged. We, therefore, have

$$\frac{m_o\, u\, \sin\theta_i}{\sqrt{1-u^2/c^2}} = \frac{m_o\, v\, \sin\theta_r}{\sqrt{1-v^2/c^2}} \tag{3}$$

or

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{v}{u}\frac{\sqrt{1-u^2/c^2}}{\sqrt{1-v^2/c^2}} \tag{4}$$

For the same data, de Broglie's theory gives[2]

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{c^2/u}{c^2/v} = \frac{v}{u} \tag{5}$$

For small u and v, equations (4) and (5) can be considered identical, but, if for example, $v = 0.96c$; $u = 0.72$ c. and $\theta_i = 60°$, then

equation (4) gives $\theta_r = 15°, 12'$; while
equation (5) gives $\theta_r = 40°, 30'$.

An experiment can be designed and θ_r can be measured. If, as is expected, the results support equation (4), de Broglie's theory needs modification. However, one must not forget the possibility that the relativistic expression for momentum may need to be revised.

(*b*) *Refraction of Kinetic Energy Waves and Matter Waves.*—In contrast to the accepted views that (i) photons travel with the same velocity c through all media and (ii) when we say that the velocity of light varies from one medium to another we refer to its group velocity and not to the photon velocity, our new postulate I in Part I leads us to assume that the photons travel through different media with velocities at present known as the group velocities of light for those media. On this basis, let a beam of matter waves or of kinetic energy waves, (briefly referred to in this section by their common name, waves) of frequency ν and therefore of energy per packet equal to $h\nu$ moving with a velocity c_1, along AB (Fig. 1) in the medium X strike LM, the surface of separation between X and Y, at an angle of incidence equal to θ_i and proceed, after refraction, with a velocity c_2 along CD at an angle of refraction equal to θ_r. According to the results obtained in section 2 above, the momentum of the waves is $h\nu/c_1$ in the medium X and $h\nu/c_2$ in the medium Y.

Since the resolved part of the momentum of the waves parallel to the surface of separation must be the same before and after refraction, therefore,

$$\frac{h\nu}{c_1}\sin\theta_i = \frac{h\nu}{c_2}\sin\theta_r \tag{6}$$

or

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{c_1}{c_2} \tag{7}$$

which is also true if the matter waves and kinetic energy waves are treated as electromagnetic waves.

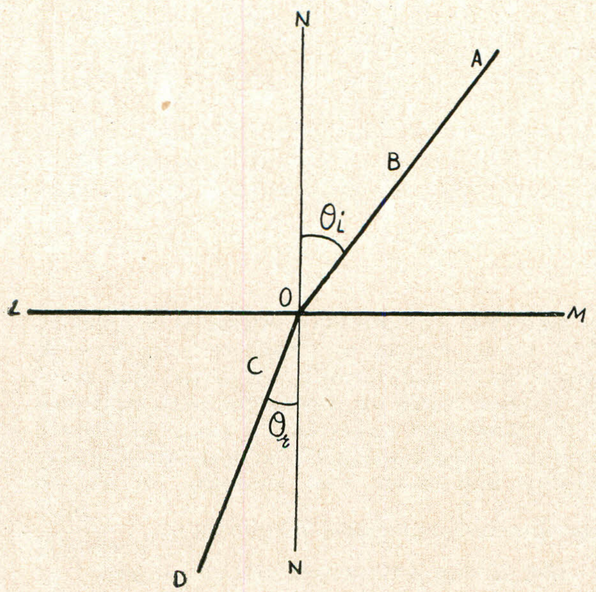

Fig. 1

It may be pointed out that, whereas the law of refraction of matter waves and kinetic energy waves is the same as that for light waves, the law of refraction for high speed electrons is found to be different. From our point of view, both the phenomena are governed by the laws of conservation of linear momentum, and the results are different because, in the case of waves, the energy of each wave packet remains unchanged as it goes from one medium to the other (with momentum change as a consequence of the change in velocity without any change in the energy of the packet as a whole), while in the case of electrons, both energy and momentum change because of their passage through the applied field.

7. Propagation of Radiation

In view of what has been said in section 4 above, radiation in transit will consist partly of the kinetic energy waves and partly of the matter waves. In the total emitted radiation of any given frequency, the energy in the form of kinetic energy waves is always greater than that in the matter wave or photon form. The former spreads out as we know light to spread out and is responsible for the phenomena known as interference and diffraction. The other form i.e. matter wave form travels in the form of photons, does not spread out like waves, and is responsible for the phenomena known as resonance radiation, absorption spectra, photo-electric effect, Compton effect, and for the characteristic distribution of energy in the black body radiation explained by Planck's radiation law. The existing theory of these phenomena is not affected by the present formulation, but it is to be emphasized that wherever absorption of radiation takes place, it is the energy in the matter wave form that we are concerned with.

Normally we should expect that since in the emitted radiation, the matter waves and the closely accompanying kinetic energy waves of the same frequency have originated from different atoms of the same substance, they are not coherent with each other; but remembering that as part of the (stationary) associated waves, matter waves are coherent with the accompanying kinetic energy waves, and knowing the fact that the photons produce interference effects, we are led to postulate:

Postulate IV.—In the process of propagation, the matter waves adjust themselves with the closely accompanying kinetic energy waves of the same frequency so as to be coherent with those waves.

Since the velocity of the matter waves and of the closely accompanying kinetic energy waves of the same frequency emitted by atoms of the same substance, is exactly the same and the laws of reflection and refraction are also exactly the same for the two, once the adjustment for coherence has taken place, it is not altered in the course of propagation. So to say, the photons ride on the kinetic energy waves of the same frequency.

8. Emission of X-Rays with the Impact of High Velocity Electrons

It is well-known that the values of h obtained from experiments on the excitation of atoms on bombardment with high speed electrons or from those designed for measurement of maximum frequency of X-rays emitted on the impact of high speed electrons of a given energy, against a heavy target, are definitely less than those obtained from other experiments. According to postulate III mentioned in section 4 earlier, it is the matter waves, and not the kinetic energy waves associated with the fast-moving electrons that are absorbed by the target. It is found that the energy associated with the matter waves calculated on the basis of the latest accepted values of h, is less than that required for exciting the atoms of the target. The value of h calculated from these experiments is therefore less than the latest accepted value. Calculations based on the application of postulate III lead to a value which is smaller still.

A little consideration shows that the process whereby a fast electron transfers energy to an atom of the target and excites it, is different from that of emission of radiation by atoms. In the case of emission of radiation, a part of the total wave energy is rejected as surplus energy. In the present case, the electron together with its associated waves, moving at a high speed, strikes the target. The atoms of the target absorb a part of its energy in the matter wave form and emit X-rays. X-rays of the highest frequency are emitted when the largest amount of energy is absorbed by the excited atoms. According to postulate III energy can be absorbed only in the matter wave form. It appears from the experimental results that the largest amount of energy which can be absorbed by an atom of the target does not consist merely of the matter waves. On the present theory, the matter waves are bound together with the kinetic-energy waves with the help of some binding energy, and the two types of waves are together bound with the mass of the electron again with the help of a binding energy. When energy is absorbed, the binding energy being also in a compact form like the matter wave energy, would be absorbed together with the matter waves. If we make use of the known values of h from other experiments, the experiments under consideration lead to the determination of the energy used in binding together the two types of associated waves corresponding to different velocities of the electron.

9. Conclusion

The formulation developed so far is based on four postulates. Ample justification has been given in Part I for postulate I, that the associated waves move with the velocity of the particle, and postulate II was derived on the analogy of the accepted relation $E = h\nu$ for a photon. These two postulates lead to the existence of two different types of associated waves and, in view of the existence of quantized atomic energy levels, postulate III stating that radiation is emitted in the form of one or other of these two types, comes as a natural and essential consequence of the first two. Postulate IV appears, at first sight, to be of the nature of an *ad hoc* assumption, since the only phenomenon that it explains is that for which it has been introduced, namely, that of the interference effects produced by photons. But when we consider that the first three postulates establish the existence, absorption, emission and propagation of photons, and that we know that photons do produce interference effects, we are directly led to postualte IV, which must therefore stand or fall with the other three postulates and not alone.

It is true that the present theory contradicts some of the important parts of de Broglie's theory which has established itself in the course of years, but as discussed in detail earlier in Part I, those parts of de Broglie's theory which are contradicted by our formulation have little theoretical justification and have never had any experimentals upport.

A possible objection against the present formulation is that it is apparently contradicted by Bucherer's experiments, which establish the relation

$$m_v = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

for high velocity electrons.[3] A careful examination of the parent equations of Bucherer's results in the light of the present formulation shows that m_v in this relation stands for the inertial mass equivalent of the associated waves and is equal to the momentum of the associated waves divided by their velocity and with this meaning of m the experimentally established relation $m_v = \frac{m_0}{\sqrt{1-v^2/c^2}}$ is valid in the present formulation as well.

The new theory makes a radical departure from the view that photons travel with the velocity of light in the empty space between molecules but have their average rate of progress retarded by the finite time consumed during the process of absorption and re-emission by the molecules.[4] The present formulation depicted in section 4 (b) above is believed to be more acceptable than the existing view referred to above.

It may be confessed that the theory seeks to introduce changes in the field of atomic physics, which may be termed revolutionary in character and if it can stand up to the criticism it is sure to meet, it should have applications of consequence even in some of those parts of atomic physics that have not been touched so far. Perhaps it could be applied to the case of a material particle falling in a gravitational field and could lead to results similar to those discussed in the present paper. There is a possibility of its application to some of the nuclear phenomena as well.

In a later communication, it is hoped further illustrations will be given of the application of this theory to, and the quantative support that it receives from, various parts of atomic physics.

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