THE CONCEPT OF MATTER WAVES, ELECTROMAGNETIC WAVES AND SCATTERING

Part I.—A Re-examination of de Broglie's Theory of Matter Waves

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The basic postulates of wave mechanics are examined critically, and it is proposed that a more satisfactory formulation can be given by combining the fundamental de Broglie postulate $\lambda = h/mv$ with two new postulates namely that the velocity of the waves associated with the particle is equal to the velocity of the particle, and their frequency ν is given uniquely by $h\nu = \frac{m_o v^2}{\sqrt{1-\frac{v^2}{c^2}}}$.

These postulates lead to the results that the total energy $E = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ is distributed between the particle and the associated waves in a manner that makes the energy localized in the particle equal to $m_o c^2 \sqrt{1-\frac{v^2}{c^2}}$, while more and more of the energy appears in the wave form as v tends to c, the velocity of light, and that the "density" of the material of a particle is invariant with respect to Lorentz transformations.

1. Introduction

Wave mechanics is based on de Broglie's theory, of which the postulates[1,2] are:

"If a co-ordinate system S is moving with a uniform velocity v relative to another co-ordinate system S* and a particle of rest mass m_o is at rest in S then

(a) an observer in S finds that a frequency ν given by

$$E = m_o c^2 = h\nu \qquad (1)$$

is associated with the particle,

(b) an observer in S* finds that a matter wave of wave length

$$\lambda^* = \frac{h}{mv} = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_o v} \qquad (2a)$$

(where h is Planck's constant), and of frequency ν^* given by

$$E = \frac{m_o c^2}{\sqrt{1-\frac{v^2}{c^2}}} = h\nu^* \qquad (2b)$$

is associated with the particle,

(c) ν of (a) above and ν^* of (b) above are connected with each other by the relation

$$\nu^* = \frac{\nu}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (3)$$

in view of the Lorentz transformations; and

(d) the phase velocity of the associated wave comes out to be $\frac{c^2}{v}$".

Several points relating to the postulates of de Broglie's theory deserve to be considered in detail, and it is the purpose of the present communication to present a re-orientation of wave mechanics in the light of such an examination.

2. Discussion of the Postulates of Wave Mechancis

(I) The relation, $\lambda^* = \frac{h}{mv} = \frac{h\sqrt{1-\frac{v^2}{c^2}}}{m_o v}$, has been quantitatively verified experimentally; but, so far as the author of this paper has been able to ascertain, there exists no direct experimental evidence to show that, for an observer in the system S, a frequency $\nu = \frac{m_o c^2}{h}$ is associated with a particle of rest mass m_o. Available theoretical evidence has the status of a postulate only.

(II) Any theory of the associated waves must necessarily ensure that these are reflected and refracted in the same direction as the particle. There is hardly any difficulty in ensuring this in the case of reflection, because reflection of a material particle (being analogous to the rebounding of an elastic ball from a hard wall) follows the same laws as those governing the reflection of waves. But, when we consider the refraction of the particle and its associated waves, we find that according to the hithertofore accepted way of looking at the phenomenon, the associated waves can, after refraction, proceed along the

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same path as the particle, only if they travel with phase velocities u nearly equal to $\frac{c^2}{v}$, where c is the velocity of light and v is the velocity of the particle in the medium concerned. Other accepted ways of dealing with the situation also lead to the conclusion that the associated waves have phase velocities in the neighbourhood of $\frac{c^2}{v}$, which is necessarily greater than the velocity of light. To get over this difficulty, it is assumed that v, the velocity of the particle, equals the group velocity of the associated waves. A closer examination reveals that this single assumption, in fact, consists of two assumptions:

(a) that more waves than one, *of nearly equal, yet distinctly unequal, wave lengths* are associated with the particle;

(b) that the medium through which the particle, and therefore also the associated group of waves, travels, is necessarily dispersive.

Assumption (a) above is exposed to the following objections: (i) the theory postulates that for a given value of v there is one and only one value of λ and therefore also of ν for the associated waves, and for this reason the assumption is apparently contrary to the theory; (ii) in dealing with the Bohr-Sommerfeld quantized orbits, we fall back to the association of a wave train of a single wave length with the electron. This is consistent with the theory but contrary to the assumption under consideration. Assumption (b) above leads to two serious difficulties, viz., (i) there is hardly any reason to believe that the medium in the interatomic spaces, through which the electrons definitely pass in the diffraction experiments, is dispersive,** and (ii) the phase velocity in the dispersive medium varies with the wave length of the associated waves so as always to give a group velocity equal to the velocity of the particle. m_o and v can both be varied simultaneously so as to get the same λ.

For example, we can have

$$\lambda = \frac{h\sqrt{1-\frac{v_1^2}{c^2}}}{(m_o)_1 v_1} = \frac{h\sqrt{1-\frac{v_2^2}{c^2}}}{(m_o)_2 v_2}$$

with $v_1 \simeq 100\, v_2$ and $(m_o)_1 \simeq \frac{(m_o)_2}{100}$ and $v_1 << c$.

We are then led to the existence of different wave groups which have the same average λ but widely different values of the group velocity. In view of the relation for group velocity, $u_g = -\lambda^2 \frac{\partial\nu}{\partial\lambda}$, this is not very plausible if the different wave groups are passing through the same medium.

(III) Since the group velocity determines the velocity with which the centre of maximum energy of the wave group travels, another difficulty arises. The associated waves consist of finite and not infinite wave trains, and if the particle is moving with a constant velocity, we should expect that no new associated waves are generated and added to it as it proceeds along its path. Since the individual waves invariably have velocities c^2/v greater than the particle velocity, they go past the particle and apparently carry their energy with them into space beyond the particle. After a finite but sufficiently long interval of time, the particle will be left without any associated waves at all; all of them will have, by that time, gone far ahead of it. There is in such a case no question of the centre of maximum energy travelling with the velocity of the particle; and the indentification of the group velocity of the associated waves with the particle velocity fails.

(IV) Let us consider further the assumption that the phase velocities of the associated waves are very nearly equal to c^2/v, which is helpful in giving an adequate explanation of refraction of a pencil of electrons as it leaves one field and enters another. But, as is shown below, acceptance of that explanation as being correct leads to a conflict with the principle of conservation of linear momentum.

Consider a pencil of electrons proceeding with a velocity u along the straight line AB (Fig. 1) in the metal box X kept at a constant potential V_1 with respect to the source of electrons. Let this beam leave the box X at the exit 'e' and enter, through the inlet 'i', another metallic box Y,

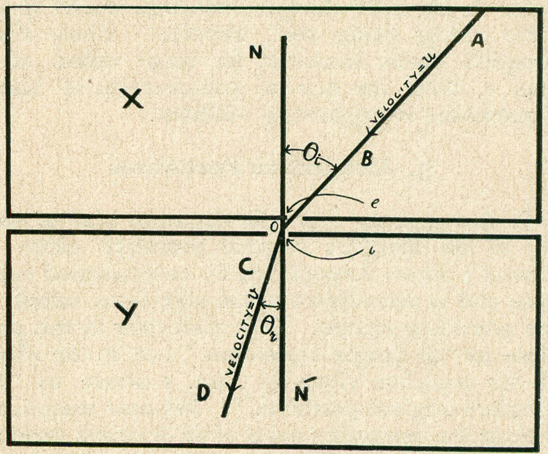

Fig. 1

** In any case, this assumption would lead to a difficult situation if there existed a nondispersive medium, and a particle moved through that medium.

kept at a constant potential V_2 with respect to the source of electrons. The adjacent walls of the two boxes are parallel to each other and the field between the two boxes is perpendicular to these walls. Let the pencil of electrons proceed in the box Y along CD with a velocity v. Let θ_i and θ_r be respectively the acute angles which AB and CD make with the direction NON′ of the field between the two boxes.

Richtmyer, Kennard and Lauritsen[3] have shown that since $v \sin \theta_r = u \sin \theta_i$ the electron waves are refracted according to the law:

$$\frac{\text{Sin } \theta_i}{\text{Sin } \theta_r} = \frac{\text{wave velocity in the medium X}}{\text{wave velocity in the medium Y}}$$

if wave velocities in the media X and Y be taken as c^2/u and c^2/v, respectively. But the relation $v \sin \theta_r = u \sin \theta_i$ is true only for small values of v and u for which $\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$ and $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ differ from unity by a negligible amount. For large values of v and u, the resolved part, perpendicular to the field, of the momentum of each electron of rest mass m_o will be $\frac{m_o u \text{ Sin } \theta_i}{\sqrt{1-u^2/c^2}}$ in the box X and $\frac{m_o v \text{ Sin } \theta_r}{\sqrt{1-v^2/c^2}}$ in the box Y; and if electron waves are still refracted according to the law,

$$\frac{\text{Sin } \theta_i}{\text{Sin } \theta_r} = \frac{\text{wave velocity in the medium X}}{\text{wave velocity in the medium Y}}$$

and their phase velocities are still to be taken as c^2/u and c^2/v, $v \text{ Sin } \theta_r = u \text{ Sin } \theta_i$ will still hold but $\frac{m_o u \text{ Sin } \theta_i}{\sqrt{1-u^2/c^2}}$ is not equal to $\frac{m_o v \text{ Sin } \theta_r}{\sqrt{1-v^2/c^2}}$ so that the resolved part of the momentum perpendicular to the field changes although there is no force acting along that direction. Thus, if de Broglie's theory is correct for large values of u and v, then the law of conservation of linear momentum is apparently violated.

3. Alternative Postulates

It thus appears that only equation 2(a) of part (b) of de Broglie's original postulate given in section 1 above is supported by experimental evidence and is not subject to the objections raised in the preceding section. (We shall refer to this part alone as "de Broglie's postulate" and to the whole of his postulate given in para. 1 above as "de Broglie's original postulate"). We now tentatively replace the remaining parts of de Broglie's original postulate by the following two new postulates:

New Postulate I.—For an observer in S*, the matter waves associated with the particle, move with the *same* velocity v as the *particle* itself.

New Postulate II.—The energy carried by a matter wave is independent of the velocity with which it moves, depends only on its frequency ν and is equal to $h\nu$, where h is Planck's constant.

The associated waves have their wave length λ^* fixed by de Broglie's relation $\lambda^* = \frac{h\sqrt{1-v^2/c^2}}{m_o v}$, and their velocity is set by the "new postulate I" at v the velocity of the particle. Once λ^* and v are determined, ν^* is determined by the well-known relation $v = \lambda^* \nu^*$ so that

$$\nu^* = \frac{v}{\lambda^*} = \frac{v.\, m_o v}{h\sqrt{1-v^2/c^2}} = \frac{m_o v^2}{h\sqrt{1-v^2/c^2}} \qquad (4)$$

Thereafter, by analogy with the relation $E = h\nu$ for a photon, the energy of the matter waves is fixed at $h\nu^*$ by the "new postulate II," which gives

$$E^* = h\nu^* = \frac{m_o v^2}{\sqrt{1-v^2/c^2}} \qquad (5)$$

4. Consequences of the New Postulates

We notice that whereas on de Broglie's theory, the energy of the moving particle is wholly in the wave form and $h\nu^*$ is equated to $\frac{m_o c^2}{\sqrt{1-v^2/c^2}}$, in the present case, on the other hand, only a part of the total energy is in the wave form and $h\nu^*$ is equated to $\frac{m_o v^2}{\sqrt{1-v^2/c^2}}$. For a given set of values of v and m_o, the ν^* of the present theory is much less than the ν^* of de Broglie's theory, but the *experimentally observed* quantity λ^* has the *same* value in either case.

According to the theory of relativity,[4] the total energy of the particle of rest mass m_o is equal to $\frac{m_o c^2}{\sqrt{1-v^2/c^2}}$ for an observer in S*. Of this total amount of energy, the part carried by its associated waves is, according to the present theory, equal to

$$E^* = \frac{m_o v^2}{\sqrt{1-v^2/c^2}} = v^2/c^2 . \frac{m_o c^2}{\sqrt{1-v^2/c^2}} \qquad (5)$$

and the remaining amount, carried by the mass is

$$\frac{m_0c^2}{\sqrt{1-v^2/c^2}} \cdot \left(1-\frac{v^2}{c^2}\right) = m_0c^2\sqrt{1-\frac{v^2}{c^2}} =$$

$$\left\{ m_0\sqrt{1-\frac{v^2}{c^2}} \right\} c^2 \qquad (6)$$

In other words, an observer in S* would find that the mass of the particle is not given by the relativistic expression $m_v = \frac{m_0}{\sqrt{1-v^2/c^2}}$ which shows an *increase* of the inertial mass equivalent of the particle with increasing velocity, but by a new expression $m_v = m_0\sqrt{1-v^2/c^2}$ which shows that the part of mass not appearing in the wave form *decreases* as velocity increases, becoming zero for $v = c$ when the *entire energy is in the wave form*. This decrease of the mass appears to be more consistent with the well-known Lorentz contraction $l^* = l\sqrt{1-v^2/c^2}$, and since there is no change of dimensions of a body in a direction perpendicular to the direction of its motion, as velocity increases, the mass and volume decrease in the same ratio so that the "density" (understood to mean "that part of the rest mass of the particle which is not appearing in the wave form, divided by the volume of that part") of the material of the particle is the same for an observer in S as that for one in S*, or for one in any other system which is inertial with respect to S and S*. This conclusion is of special significance, because it means that the "density" of the material of a particle is invariant with respect to Lorentz transformations.

A detailed discussion of the deductions from the new postulates as applied to the theory of reflection, refraction, scattering, etc., will form the subject matter of a later communication.

References

1. De Broglie, Phil. Mag., **47**, 448 (1924).
2. Thomson and Cockrane, *Theory and Practice of Electron Diffraction.*
3. Richtmyer, Kennard and Lauritsen, *Introduction to Modern Physics.*
4. Bergmann, *Introduction to the Theory of Relativity.*