STATISTICAL CONTROL OF LOG BREAKDOWN*

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(Received May 10, 1961)

Careful attention to the manner and precision of sawing logs into lumber has been frequently and strenuously advocated by leading lumber specialists of scientific and industrial organizations. This advocacy has had as its purpose the maximizing of the yields of the higher grades of lumber. Since these higher grades of lumber are almost invariably in short supply, the consequent demand for these items is good and prices are normally high. The thought and energy expended in presenting these pleas for better milling procedures has intensified the awareness of the problems associated with decreased log size and lower yield values of these logs. It certainly is good economics to obtain the highest yield of the upper grades of lumber consistent with maintaining a reasonable production rate. In particular Telford,[9] Smith,[8] Brown and Bethel,[4] and others have emphasized the necessity of using correct methods of sawing for maximizing the yields of the higher grades of lumber. Bethel and Barefoot[3] indicate the sawyer's job is one of those prime positions where the decisions of one man, properly made, are extremely vital to the economic well being of a lumber manufacturing operation. The application of statistical quality control procedures to grade recovery of lumber which would be comparable to those used in controlling dimensions could be extremely valuable to the sawmilling industry.

No systematized mathematical approaches have been developed for evaluating the breakdown of logs into lumber. Statistical quality control procedures, such as Shewart's original p-chart[7] have not been applied to log breakdown because of the complexities arising from the number of log grades, the number of lumber grades, and the slowness with which data could be collected and interpreted. There being a definite need for controlling the processes concerned with log breakdown, this study was undertaken primarily to provide a systematic method of control in evaluating the correctness of log sawing procedures for single logs as well as groups of logs. A simple system would require the adoption of a statistical procedure which could be readily computed, pictorially presented, and easily interpreted for controlling the percentages of lumber yields by grades from logs.

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The opinion existed that the p-charts and Chi-square charts were not desirable for solving the problem of controlling grade yields. The objection to using the p-chart for logs concerns the necessity for maintaining a chart for each lumber grade with each log grade; for all logs sawn a collective picture for immediate evaluation or control is difficult to obtain from these unrelated charts. On the other hand, the Chi-square chart could be adapted to combine all the yields of all the logs on one chart but it lacks simplicity of interpretation since not only a constant sample size is required but the chart does not directly evaluate the quality in question. This study was undertaken for developing a control procedure involving many percentages as found in lumber yields at a sawmill.

Procedures of the Study

The initial work of the study centered about developing a trial control procedure applicable to the lumber yields of logs. The criteria for such a method were these:

(1) One chart was desirable for presenting the results of sawing all logs regardless of their grade.

(2) Computations should be fairly simple, direct, and understandable.

(3) The control chart should provide the means for ferreting out the assignable causes associated with wrong cutting procedures.

(4) If possible, Shewart's $\bar{X}$ and R charts were to be the basis of the control procedure used.

Under these criteria an analysis method was outlined and used at two saw mills cutting yellow poplar (*Liriodendron tulipifera* L.) lumber. At these mills the randomly selected logs were scaled, graded into No. 1, No. 2, No. 3, and No. 4 logs and observed during sawing for any unusual occurrences. Following cutting, the lumber from the logs was graded into the standard hardwood lumber grades for yellow poplar and tallied. The resulting data were plotted on charts (Figs. 1 and 2) according to the first of the schemes presented in the next section. Subsequent study of these chart led to their rejection as a satisfactory means o controlling the grade recovery of hardwood logs. The second scheme, with its charts (Figs. 3 to 7) presented in this report was then adopted for trial. All the results were analyzed as to their effectiveness in controlling log breakdown.

The Mathematical Procedures

The first approach to controlling grade recovery was made through a set of indexes plotted on $\bar{X}$ and R charts; the second procedure involved a refinement of the p-chart. Both plans made use of the data generated in the study as follows:

(1) Standard indexes, ϕ 1, were computed from data in the Forest Products Laboratory log grade report[1] for No. 1, No. 2 and No. 3 logs. For No. 4 logs this index was computed from unpublished data.[5] The standard value ϕ 1, was selected for each log grade by finding that percentage of yield of a lumber grade or combination of grades which approached 0.50. For example, from No. 1 yellow poplar logs the proportion of No. 1 Common lumber expected is stated to be 0.457 (1). Arbitrarily then $\phi\ 1 = 0.457$. Appropriate selections gave:

a. No. 1 logs proportion of No. 1 Common lumber $= 0.457 = \phi\ 1$
b. No. 2 logs porportion of F & S and No. 1 Common lumber $= 0.466 = \phi\ 2$
c. No. 3 logs, proportion of No. 1 Common and 2-A lumber $= 0.491 = \phi\ 3$
d. No. 4 logs, proportion of 1-A and 2-B lumber $= 0.553 = \phi\ 4$.

(2) For each log graded, sawed, and tallied by lumber grade a quantity

$$B_j = \frac{P_j}{\phi_i}$$

was computed where Pj was the observed proportion of lumber from the jth log falling into the same classification used in selecting ϕ_i for that grade of logs. For example, a No. 1 log producing a total 180 board feet of lumber and having 100 board feet of No. 1 Common lumber would have a B1 value as below:

$$B_1 = 0.556/0.457 = 1.217$$

(3) At this step the two procedures used to analyze the data diverge and the results of plotting the charts are quite different. The assumptions concerning the approximate variances will be developed as each procedure is further explained.

In the first proposal the normal distribution

Table 1.—Sample Data Sheet for Bi's.

Sub-group	B j Observations				—	R
1	1.11	0.35	0.63	0.93	0.75	0.76
2	1.60	0.69	0.83	1.61	1.18	0.92
:	:	:	:	:	:	:
:	:	:	:	:	:	:
N	1.79	0.25	0.00	0.88	0.73	1.79

was assumed to describe the averages of a set of Bj's. For this reason the data was subgrouped into sets of four Bj's; $\bar{X}$ and R charts were then used as in standard Statistical Quality Control analysis. The Bj's were recorded and statistics computed for the data of two sawmills as illustrated in Table 1.

There seeming to exist a situation involving "too-good" control, several simple subgrouping variations of these data were tried for interpreting the data. There being nothing suggesting that a new subgrouping method would solve the situation of "too-good" control (Figs. 1 and 2) of the above procedure, a new P-1 charting technique, the second approach adapted, was formulated and scrutinized. Since this control charting procedure seems to hold some merit for use in other that log and lumber studies, the theory of the P-1 control chart is presented in more detail.

For the purposes of pictorial presentation of percentage figures and their appropriate three sigma limits Shewart[7] suggested the use of a p-chart. This chart was based on the binomial distribution of statistical theory and is fully explained in Grant's[6] text. Briefly the theory is based upon dividing a given population of things into two groups, A and B, and stating the resulting division in terms of proportions thusly:

$$P_A = \frac{\text{Number of A objects}}{\text{Total objects}} \qquad (1)$$

or

$$P_B = \frac{\text{Number of B objects}}{\text{Total objects}} \qquad (2)$$

where P_A or P_B is the proportion of A objects or B objects to the total number of A and B objects. The variance of sample proportions can be shown to be.:

$$V(p) = \frac{p\,q}{n} \qquad (3)$$

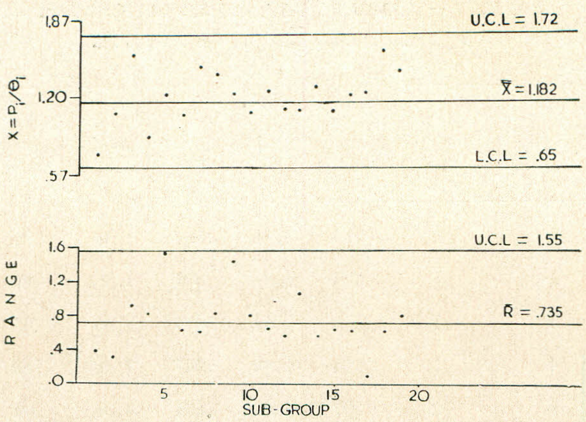

Fig. 1.—X and R Control Chart For Mill A.

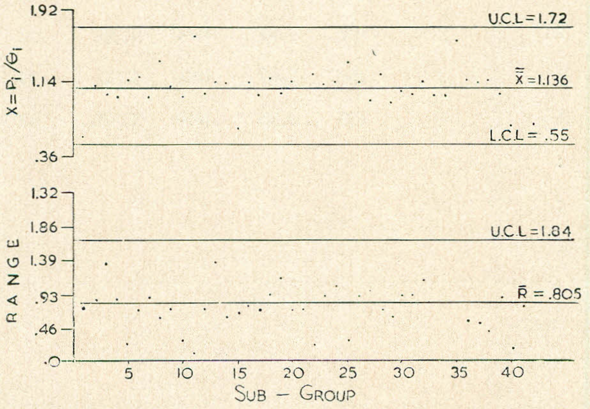

Fig. 2.—X and R Control Chart for Mill B.

where n = number of objects in a sample,
p = proportion of a given object in a sample of size n, q = 1 − p

The p-chart for control purposes then is calculated and plotted according to the ASTM Manual[2] with limits equal to

$$p \pm 3 \sqrt{\frac{\bar{p}\ \bar{q}}{n}} \qquad (4)$$

where p is an average fractional proportion of many samples and n is the number of a given sample for which one value of p is computed.

It is obvious from an examination of the Forest Products Laboratory's work[4] that within each log grade, yields of each lumber grade vary from the corresponding yields of the other log grades, for example, the percentage yield of No. 1 Common lumber of a given species of logs from No. 1, No. 2 and No. 3 logs might be represented by $\overline{p_1}$, $\overline{p_2}$, and $\overline{p_3}$ respectively. Three charts would be required if p-charts were used for control purposes. However, if the stadnard yield, ϕ, as developed in paragraph (1) of this section of a given grade of lumber is established for a given log grade such that B_j has the expected value of 1.0

$$E(B_j) = E\frac{P_j}{\phi_1} = 1.00 \qquad (5)$$

then the possibility of incorporating P_1, P_2, and P_3 into one chart exists since $E(B_1)$, $E(B_2)$ and $E(B_3)$ should all have the same mean. Therefore, we may calculate by summing over all grades, the quality

$$\bar{B} = \frac{E(B_j)}{\text{Number of samples}} \qquad (6)$$

The selection of the appropriate variance remains. Since the variance of p is

$$V(p) = \frac{p\ q}{n} \text{ and}$$

$$V(\phi) = 0 \text{ by definition}$$

then
$$V\left(\frac{P}{\phi}\right) = \frac{p\ \ q}{n\ \phi^2} \qquad (7)$$

Letting $\frac{p\ \ q}{\phi 2} = K^2$ then

$$V\left(\frac{P}{\phi}\right) = \frac{K}{n} \qquad (8)$$

As $\phi \rightarrow 0.50$ then $E(P) \rightarrow 0.50$ (from 5) and

$$V\left(\frac{P}{\phi}\right) \rightarrow \frac{1}{n} \qquad (9)$$

It is not necessary that the ϕ's approach 0.50 or that they all be of the same value but if they are so chosen as to be practically equal and nearly 0.50, then the greatest variation in a control chart's limits will be caused by varying sample size. As will be shown, this variability of the limits can all but be ignored in working with logs and selected lumber yields. Selection in this manner also provides the narrowest limits possible.

Following this theory a p-1 control chart can be plotted with the values from entirely different origins, i.e., logs of different grades. Letting the central line equal $\bar{B}$, then the limits are defined as:

$$\text{Control limits} = \bar{B} \pm 3\sqrt{\frac{K}{N}} = \bar{B} \pm 3\,s \quad (10)$$

As is the usual procedure with SQC charts any point falling outside a three sigma limit line is investigated for the presence of an assignable cause. The points which are out of control may also yield information on the origin of the trouble as do similar points on the $\bar{X}$ and R charts. This feature overcomes the objection to the Chi-square chart which merely records the evidence of agreement to some standard. Points falling consistently high or low also aid in controlling the process or in improving the process when careful and correct decision-making is evidenced. P-1 charts were prepared as above for each of the five saw-mills selected for study. These mills were located in the Piedmont and mountains of North Carolina. Due to the circumstances of collecting the data, very little experimentation on sawing order or procedures could be practised or effectively suggested; therefore, the charts presented merely reveal the history of each operation with no corrective actions recorded. The p-1 chart for each mill was then drawn according to the following practice using formula 10:

(1) $\bar{B}$ = central line

(2) Upper and lower control limits.

a. For logs of grade 1 : $\bar{B} \pm \frac{3\,(1.09)}{\sqrt{n}}$

b. For logs of grade 2 : $\bar{B} \pm \frac{3\,(1.07)}{\sqrt{n}}$

c. For logs of grade 3 : $\bar{B} \pm \frac{3\,(1.02)}{\sqrt{n}}$

d. For logs of grade 4 : $\bar{B} \pm \frac{3\,(.899)}{\sqrt{n}}$

The letter, n, is the total board footage yield of all grades of lumber from a given log.

Table 2 gives the calculations for the exact control limits for the first ten points of Fig. 3. The value for $\bar{B}$ in Fig. 3 and Table 2 is based on data from the first 48 logs studied at Mill A.

TABLE 2.—SAMPLE DATA COMPUTATIONS FOR P-1 CONTROL CHART OF MILL A.

Log Number	Log Grade	Board Footage Yield n	Bj		Upper Control Limit	Lower Control Limit
1	1	263	1.11	.067	1.34	.94
2	1	208	0.35	.076	1.36	.91
3	1	223	0.63	.073	1.36	.92
4	1	156	0.93	.087	1.40	.88
5	2	227	1.60	.071	1.35	.92
6	1	232	0.69	.072	1.35	.92
7	1	248	0.83	.069	1.34	.93
8	2	187	1.61	.078	1.36	.91
9	2	282	1.67	.064	1.33	.94
10	4	103	0.33	.089	1.40	.87
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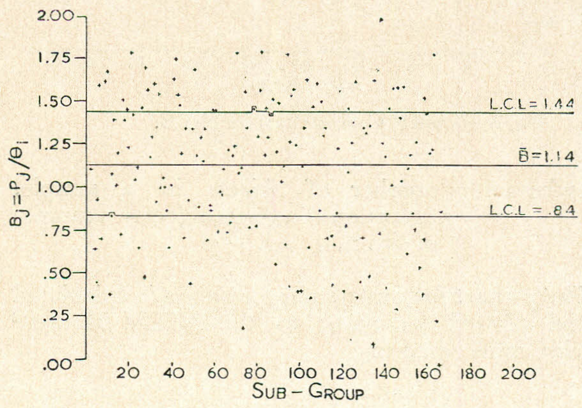

Fig. 3.—Pl Chart For Mill B.

Generally a preliminary $\bar{B}$ may be safely calculated from the first 20-25 logs.

$$B_j = \frac{P_j}{\phi_{1,2,\text{ or } 3}}$$

$$\bar{B} = \frac{\overset{48}{\underset{j=1}{E}} B_j}{48} = 1.136$$

Control limits $= \bar{B} \pm 3\,s$; formula 10

Trial control limits for the P-1 chart may be derived for all except questionable points by using these empirical rules for the selection of n in formula 10:

(1) For logs which will average 100-135 board feet, substract 13 from the average and use a value of $n = 87$-122 accordingly.

(2) For logs which will average 60-80 board feet, substract 12 from the average and use a value of $n = 48$-68 accordingly.

(3) For logs which will average about 45-60 board feet, substract 11 from the average and use a value of $n = 34$-49 accordingly.

Trial limits based on the above procedure are shown on the charts of Figs. 3-7 and are corrected for some of the questionable points as shown. These corrections follow the standard practices used when a p-chart having variable samples sizes is plotted with constant trial limits.

Interpretation of Results

A rather casual examination of Fig. 1 would lead a quality control technician to exclaim with joy, 'At last a process immediately in control.' Yet, with Fig. 2, which represents another mill's process and this too is immediately 'in control' the same technician should suspect that some errors in assumptions have been made with respect to the charting technique in use. For it is almost axiomatic that no manufacturing process is ever completely and always in statistical control. Further, almost no process approaches a state of being in control when first studied. For this reason careful attention was turned upon these charts; different subgroupings were tried, but basically the same conclusion was reached in each case. The technique used in plotting Figs. 1 and 2 resulted in charts which showed a state of 'too good' control. Obviously, the variation within the subgroups was too large for use in computing the three-sigma limits

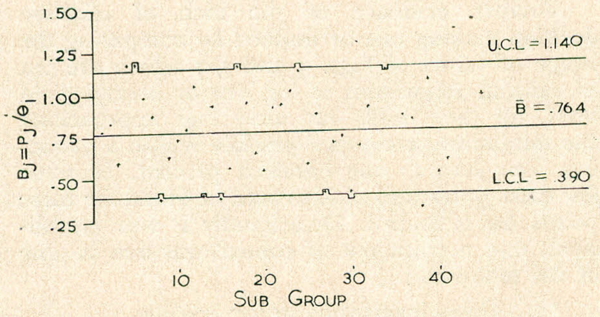

Fig. 4.—Pl Chart for Mill C.

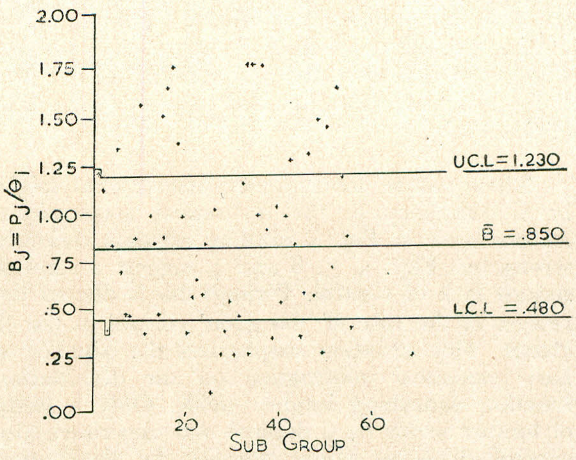

Fig. 5.—Pl Chart for Mill D.

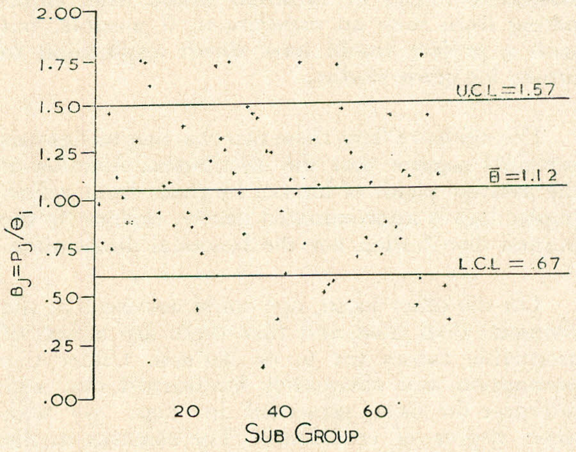

Fig. 6.—Pl Chart For Mill E.

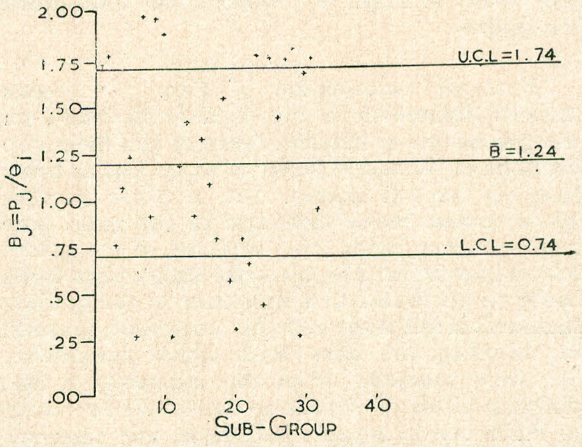

Fig. 7.—Pl Chart for Mill A.

of an $\bar{X}$ and R chart. Perhaps arbitrarily the selection of two sigma limits would solve this problem for a practical operation. But this approach leaves much to be desired since these limits could still give false assurances of having a well organized cutting procedure when in fact such may not be. For this reason attention was directed to the new P-1 charts for a more detailed analysis of the data of this study.

These charts, too however, reveal what might be a deficiency in the theoretical assumptions. As observed in Fig. 4 nothing appears seriously wrong but Figs. 3, 5, 6 and 7 suggest a state of serious lack of control. Perhaps this is due to the reverse of the case of 'too good' control in the $\bar{X}$ charts. This situation emphasizes the absence of basic statistical information on the distribution function describing lumber yields. Only a study of lumber yields by a defined and experimentally controlled sawing would establish the distribution function describing these yields from logs. All the assignable causes of variation would need eliminating from such an experiment, if possible, but such a project would have direct application to future log grade studies.

The evidence from these two charting techniques certainly suggest that the distribution function of the average yields of lumber by grade from yellow poplar logs is approximated neatly, neither by the normal distribution nor the binomial distribution.

On the other hand, one could assume that the binomial does hold and that there are a host of assignable causes yet to be defined. Under this assumption and adequately forewarned, one can interpret the data presented in Figs. 3-7 with some degree of confidence. The remarks in the remainder of this section will be confined to the charts of Figs. 3-7 under the forewarned assumption.

A casual examination of Figs. 3-7 reveals obvious differences in the yield levels based on the parameters as selected. Some of this difference could be attributed to better or worse logs as taken from any tract of timber. Yet, a closer scrutiny of the charts shows that four of the mills were getting higher yields than expected of the grades selected for study. The generally higher yield levels could be attributed to a difference in the grading standards of this study and the standards imposed in collecting the data from which parameters, ϕ_i, were selected. If so, the indication is that differing yields of various studies could be traced to difference in human judgement and interpretation of grade rules.

Two other examinations should also be made, The differing yield levels of the five mills may have been due to different sawyers or to various sources of logs or to a combination of these two variables. There are available no quantitative data substantiating the thesis that yields within a log grade vary with locations but neither is there any such data on sawyers. Before precise statements could be made, these two items would need definitive study. It is, however, much easier to subscribe to the theory that the sawyers have much more influence on lumber yields than the source of a given set of logs. Therefore, until evidence to the contrary is presented, it is safe to assume that sawyers, in making decisions on sawing procedures, do exert an influence on yields. The lack of control evidence in the charts of this study suggests, therefore, that they are not doing a consistent job.

Log size, as such, would appear from these studies to have little affect on the state of control. The logs on which Fig. 3 was based averaged 127 board feet in size. The operation was evidently out of statistical control, whereas the logs on which Figs. 4 and 5 were based averaged 94 and 85 board feet respectively. The mill which is represented by Fig. 4 seemed to be the best controlled mill found in the study, while the mill of Fig. 5 was among the worst. It would appear then that log size alone cannot be used in explaining the lack of control or a state of better control.

Peculiarly enough, as noted in an earlier paragraph, the data of Fig. 4 indicates a low level yield of those grades chosen for control purposes. Tracing this information back to the original data revealed that more of the lumber which was produced in this mill was of higher grades than expected. This mill actually was recovering more of the higher grades of lumber than any of the other mills. Again, one must ask, 'Which is responsible, the sawyer or the source of the logs?' No satisfactory evidence is available. It can be pointed out, however, that the mills of Figs. 4 and 5 were located within a few miles of each other.

Further evidence of the lack of effective decision making can be implied by comparing the range of values of Figs. 3 and 5-7 which are out of control. Regardless of log size or overall yield level, all these mills cut some logs approaching the minimum percentage possible within a grouping, i.e., zero to one hundred percent. In some few cases assignable causes could be found for these points out of control. Occasionally it was established that a point out of control was due to one of the following:

(1) Gross inaccuracy in log grading.
(2) Incorrect sawing procedures.

(3) Decayed zones in the interior of solid-looking logs or other hidden defects of broad scope in the log.

(4) Borderline decisions about the log grade. Usually a combination of 2 and 3 was cited as a basic cause. This only, though, because the sawyers usually paid little attention to turning for grade.

The lack of precision in the log grades, lumber grades, sawing decisions and grading decisions probably contribute heavily to the variability recorded. In terms of the present system of classifying yellow poplar logs and lumber yields from those logs, the variability of the data is extremely large and unexplainable. Quite evidently much basic information needs gathering through totally new approaches such as are now being used by several laboratories.

Disregarding the base source of data one can still place some confidence in the P-1 charting technique presented in the paper. Given a set of controllable conditions, the method would appear to be workable; certainly Fig. 4 would lend support to this conviction. A fair trial, using data from sources other than logs and conducted under better conditions, would be in order.

Summary and Conclusions

Any true summary of this study must point out the failure to control the grade yields of lumber from yellow poplar logs using the techniques worked out during the course of the research. Certainly, any attempt at controlling these yields is handicapped by our present limited knowledge about predicting the interior appearance of a log and the resulting yields of lumber by grades.

The use of a control charting technique comparable to those used in controlling dimensions did not prove acceptable. These charts immediately suggested a state of 'too good' control. They were abandoned as effective means of controlling the grade recovery of lumber in logs.

The development of the P-1 charts seems valid in theory and in one mill, of the five studied, evidence of its usefulness is presented. The other four mills are shown as being widely out of control on the P-1 charts with respect to the expected yields of lumber from logs of a known grade. Some reasons for this state of great variability are given in the body of the report and are the basis for these conclusions:

(1) An accurate and precise prediction of the grade yields of individual yellow poplar logs is not predictable through use of the present control systems.

(2) The P-1 chart is a feasible charting technique for combining several percentages of interest into one master percentage control chart.

(3) As shown by the P-1 chart the level of grade yields of yellow poplar varies from mill to mill. There is no specific data suggesting this is a difference attributable either to the sawyer or to the source of the logs.

(4) Log size in itself does not appear to influence the state of control existing at a given mill.

(5) Of the five mills studied, only one approached a state of statistical control when using the new conrtol technique.

These conclusions and the study firmly suggest the dire need for some imaginative and energetic research directed toward finding a mean of predicting the internal appearance of a log. Perhaps, the basic distribution of lumber yields by grades within logs of a given grade should be established. The investigations of this study suggest that of the two basic theoretical distributions assumed and used, neither adequately described the data to the desired degree of precision. Without a further basic study for identifying and eliminating assignable causes in the practice of sawing logs, there can be no effective statistical control procedures applied to this all important decision making step in the manufacture of lumber.

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