ROOF SLABS IN LOW-COST HOUSES

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This paper describes the design and testing of a roof in which the flexural strength of the concrete has been used to minimize the use of steel (from 170 lb. to 34 lb. for a room of 10′ × 11′ inside dimensions). This reduces the cost considerably without affecting the comfort in the house. The high test load indicates that such roof can also be used for more than one storey buildings.

Introduction

Roofs have been given much attention in the construction of low-cost houses, as these involve, to a greater extent, the use of mild steel, at present an imported material. Economy has been effected in the use of steel and cement, either by reducing the thickness of the roof slab through improving the quality of concrete (making it more dense), or by laying plain concrete tiles over precast reinforced concrete battens. Another approach, using prestressed concrete planks, has been approved for refugee houses, and a factory for the production of such precast units on a large scale is going to be set up in Karachi very soon.

The above modifications have been based simply on economy in construction, and the important factor of thermal comfort was totally negelcted. Thus, it was found that through reduction in the thickness of slab and by making the concrete more dense, the thermal conductivity of the roofing unit increased considerably. As a result, the houses were very warm in the summer and cool in the winter, and consequently they were uncomfortable for a major portion of the year. The use of an additional insulation layer on top of such roof slabs is not feasible due to the extra expense in the low-cost houses. An attempt has therefore been made by the author to design and test a type of roofing that can cut down the cost without affecting comfort in the house. The design of the roof, which has been discussed in the following, reduces the quantity of steel from approximately 170 lbs. for the construction of a single room of 10′ × 11′ inside dimensions to about 34 lbs. without any change in the quantity of concrete. In an earlier communication[1] on low-cost roofing for large-scale production of small houses, it has been mentioned that the precast units need a proper water-proofing layer. As the cost of this water-proofing layer is considerable, the roof has been designed as cast-in-situ to eliminate this, and it needs approximately the same amount of centering as in the conventional system. The constructional details are given in Fig. 1.

Design

In a reinforced concrete slab, the strength of cement concrete is generally neglected on the tension side and mild steel bars are provided to take the entire tensile stresses. In the case of this design, the flexural strength of the concrete has been utilized to reduce the amount of steel required on the tension side of a slab. The roof (10′ × 11′ inside) has been divided into three spans of plain concrete sections by providing two reinforced concrete beams projecting 6″ × 6″ below the ceiling level. As the beams have been designed on a rectangular section, these projections can be placed on the top of the roof if so desired, thus improving the appearance of the ceiling.

For calculating the amount of the stresses induced in the slab and beams, let M_A, M_B, M_C & M_D denote bending moments in foot pounds and R_A, R_B, R_C & R_D reactions in pounds at the supports A, B, C and D, respectively (Fig. 2); A and D are wall supports while B and C are beams, l_1 and l_2 being the effective spans of the slab in feet. W is the total weight (including superimposed load) in pounds per square foot area of the slab.

$$M_A = M_D = O \text{ and } M_B = M_C$$

$$2l_1 + l_2 = 10.5 \text{ ft.}$$

Applying three moments theorem, we get

$$M_A.l_1 + 2(l_1+l_2)M_B + M_C.l_2 = \frac{w}{4}(l^3{}_1 + l^3{}_2)$$

$$\text{or } M_B = \frac{w}{4}\frac{(l^3{}_1 + l^3{}_2)}{(2l_1 + 3l_2)} \text{ lbs. ft.} \quad (1)$$

Maximum positive moment in the middle span at centre

$$= \left(\frac{wl^2{}_2}{8} - M_B\right) \text{ lbs. ft.} \quad (2)$$

Let M_X be the positive moment in the end span at any section XX′ at a distance X from A (Fig. 2)

$$\therefore M_X = \left(\frac{wl_1 x}{2} - \frac{wx^2}{2} \right) - M_B \frac{x}{l_1}$$

For maximum moment, differentiating the above equation we get

$$x = \frac{l_1}{2} - \frac{M_B}{wl_1} \qquad (3)$$

By substituting the value of x in the above equation of M_X, the value of maximum positive moment can be obtained.

To find out the reaction of the slab, we get

$$R_A = R_D \text{ and } R_B = R_C$$

$$\text{and } 2(R_A + R_B) = (2l_1 + l_2) \text{ W lbs.}$$

$$= 10.5 \text{ W lbs.}$$

Now taking moments about B, we get

$$- M_B = R_A l_1 - \frac{wl^2{}_1}{2}$$

$$\text{or } R_A = \frac{Wl_1}{2} - \frac{M_B}{l_1} = \text{Wx lbs.} \qquad (4)$$

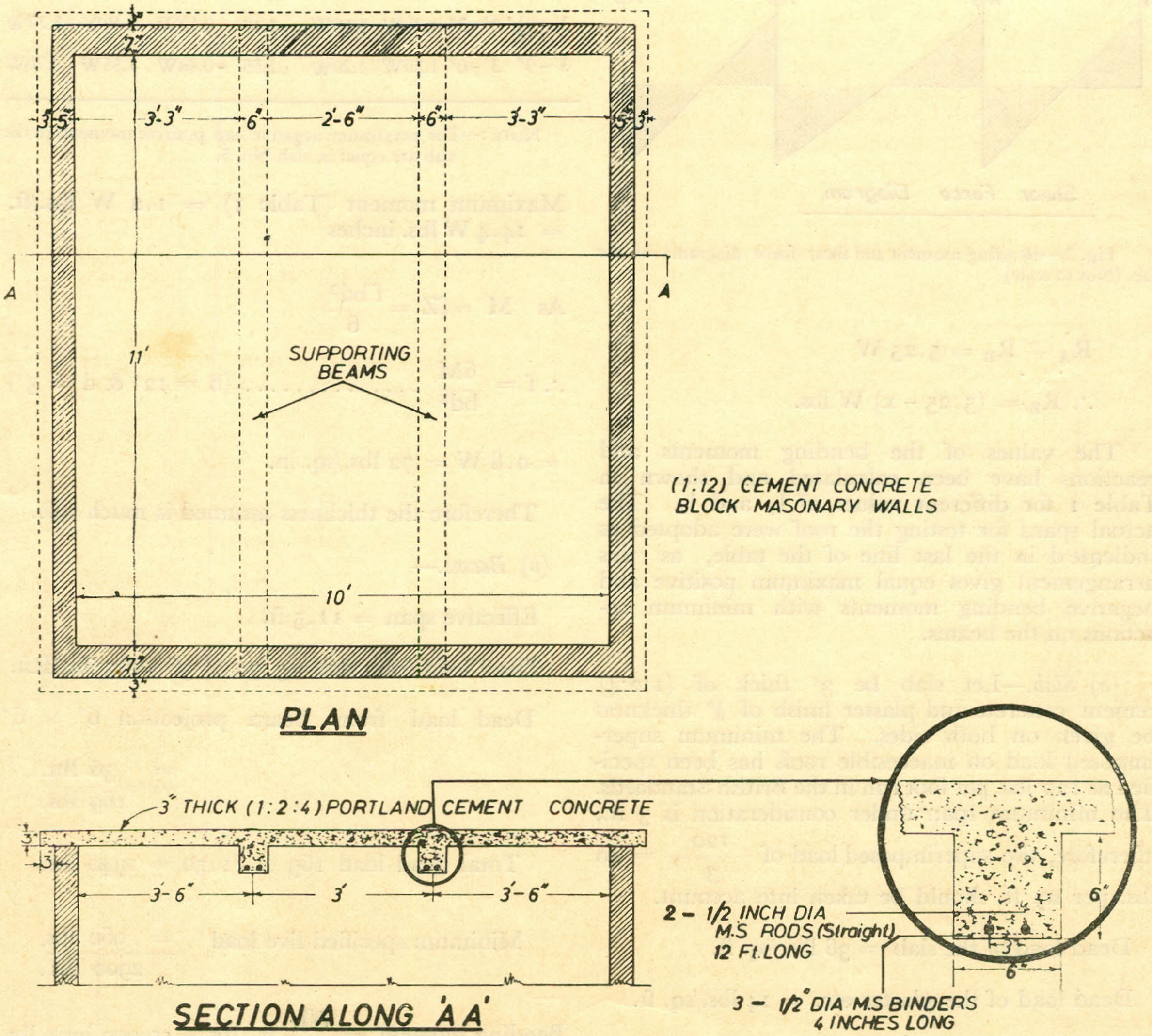

Fig. 1.—The constructional details of room with low-cost slab roof.

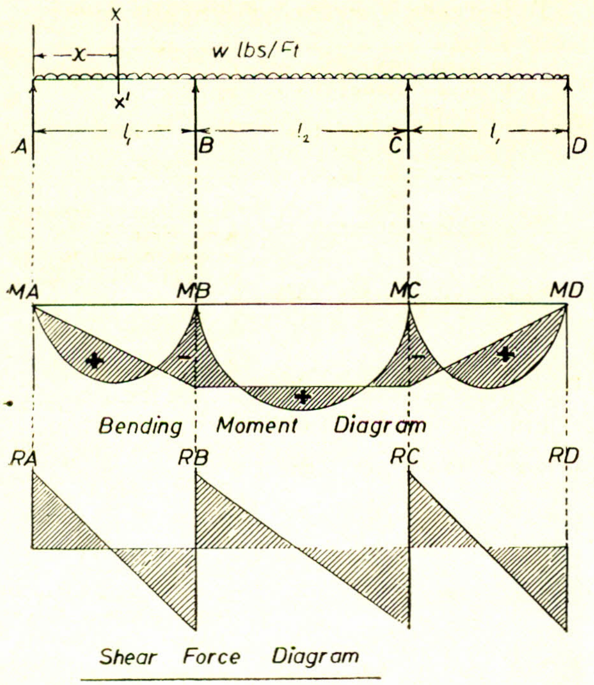

Fig. 2.—Bending moment and shear force diagrams of the slab. (Not to scale)

$$R_A + R_B = 5.25\ W$$

$$\therefore R_B = (5.25 - x)\ W \text{ lbs.}$$

The values of the bending moments and reactions have been calculated and shown in Table 1 for different values of l_1 and l_2. The actual spans for testing the roof were adopted as indicated in the last line of the table, as this arrangement gives equal maximum positive and negative bending moments with minimum reactions on the beams.

(*a*) *Slab.*—Let slab be 3″ thick of (1:2:4) cement concrete and plaster finish of ½″ thickness be given on both sides. The minimum superimposed load on inaccesible roofs has been specified as 120 lbs. per foot run in the British Standards. The minimum span under consideration is 3 ft., therefore, the superimposed load of $\frac{120}{3} = 40$ lbs. per sq. ft. should be taken into account.

Dead load of the slab = 36 lbs./sq. ft.

Dead load of the plaster etc. = 14 lbs./sq. ft.

Total load (w) = 90 lbs./sq. ft.

TABLE 1.—BENDING MOMENTS IN THE SLAB AND REACTIONS ON THE WALLS AND BEAMS.

Effective span l_1	Effective span l_2	Bending moment M_B (in. ft. lbs.)	Maximum positive moment in the end span (in ft. lbs).	x = Distance of the moment in column (5) from end A (in ft.)	Maximum positive moment in the middle span (in ft. lbs.)	$R_A = R_D$ = Reactions on 5″ thick walls (in lbs).	$R_B = R_C$ = Reactions on beams (in lbs.)
3′-0″	4′-6″	1.50W	0.50W	1.00	1.03W	1.00W	4.25W
3′-3″	4′-0″	1.32W	0.74W	1.22	0.68W	1.22W	4.03W
3′-6″	3′-6″	1.23W	0.98W	1.40	0.31W	1.40W	3.85W
3′-7½″	3′-3″	1.20W	1.10W	1.48	0.12W	1.48W	3.77W
3′-9″	3′-0″	1.20W	1.20W	1.55	-0.08W	1.55W	3.70W

NOTE:—The maximum negative and positive moments in the slab are equal in slab No. 5.

Maximum moment (Table 1) = 1.2 W lbs./ft.
= 14.4 W lbs. inches

$$\text{As} \quad M = fZ = \frac{f\,bd^2}{6}$$

$$\therefore f = \frac{6M}{bd^2} \text{ .... } (b = 12'' \ \&\ d = 3'')$$

= 0.8 W = 72 lbs./sq. in.

Therefore the thickness assumed is much safe.

(*b*) *Beams.*—

Effective span = 11.5 ft.

Dead load from slab = 3.70 × 36 = 133 lbs./Rft.

Dead load from beam projection 6″ × 6″
= 36 lbs.
169 lbs.

Total dead load 169 × 11.50 = 1940 lbs.

Minimum specified live load = 960 lbs.
2900 lbs.

$$\text{Bending moment} = \frac{WL}{8} \text{ ft. lbs.} = 50{,}000 \text{ inch lbs.}$$

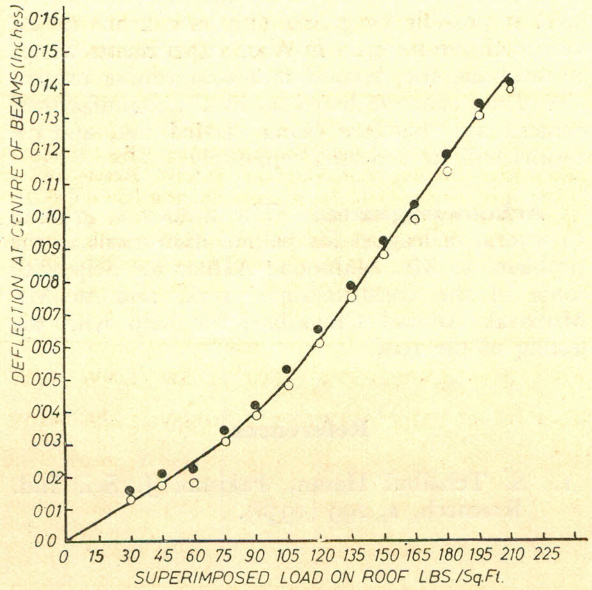

Fig. 3.—Observed relation between super imposed uniformly distributed load on the roof and deflection under beams at centre.

Taking the beam as of rectangular section, the stresses in steel 18,000 lbs. per sq. inch, the maximum stress in concrete comes to 760 lbs./sq. inch. Two bars of ½″ dia. are found sufficient. The beam is safe in shear and bond stresses.

Testing under Uniformly Distributed Load

The roof was laid in (1:2:4) cement, Malir sand and double screened coarse aggregate (Fig. 1) from Malir River bed. The water cement ratio (which was inclusive of the water absorbed by the dry aggregate) was kept at 0.6, as this is generally specified on Government works. The centering was kept in place for the normal period and then removed. The ceiling and roof were then plastered with (1:6) cement-sand mortar. After 56 days, the roof was progressively loaded with sand bags as shown in Table 2. The superimposed load was increased by 15 lbs. after every 48 hours except in two cases when the superimposed load were kept unchanged for 72 hours. Deflections were recorded under the beams at centre and plotted against loads (Fig. 3). These tests were conducted by Mr. Mubarak Ahmad Choudhari.

The deflection gauges were removed at 210 lbs. load and the beams were supported with wooden planks from below. The superimposed load was again increased by 15 lbs. per sq. ft. of the roof after every 48 hours. Cracks a ppeared approximately at the point where maximum positive moment occurs, and were first noted at a live

TABLE 2.—DEFLECTION TESTING OF THE DESIGNED ROOF UNDER VARYING SUPERIMPOSED LOAD.

Date	Total superimposed load in lbs./sq.ft.	Deflection at centre in inches: Beam No. 1	Deflection at centre in inches: Beam No. 2	Remarks
8.10.60	30	.000	.000	The readings of deflection were taken before increasing the loading of the roof to the next value.
10.10.60	30	.008	.010	
11.10.60	45	.015	.013	
12.10.60	45	.016	.013	
13.10.60	60	.021	.017	
14.10.60	60	.022	.018	
15.10.60	75	.022	.018	
16.10.60	75	.026	.022	
18.10.60	90	.034	.031	The superimposed load of 75 lbs./sq. ft. was allowed to remain for 72 hours.
19.10.60	90	.036	.033	
20.10.60	105	.042	.039	
21.10.60	105	.046	.042	
22.10.60	120	.053	.048	
23.10.60	120	.054	.050	A horizontal crack appeared all round in the supporting walls at about 9″ below the ceiling level.
24.10.60	135	.065	.061	
25.10.60	135	.068	.064	
26.10.60	150	.079	.075	
27.10.60	150	.085	.081	
29.10.60	165	.092	.088	The superimposed load of 150 lbs./sq. ft. was all wed to remain for 72 hours.
30.10.60	165	.095	.090	
31.10.60	180	.103	.099	
1.11.60	180	.108	.104	
2.11.60	195	.118	.113	
3.11.60	195	.120	.115	Fine cracks appeared in the middle of the beams.
4.11.60	210	.133	.130	
5.11.60	210	.140	.138	The deflection was noted after 24 hours of 210 lbs. superimposed load.

load of 255 lbs./sq. ft. The loading on the roof was continued till 360 lbs./sq. ft. was reached., At this load, the cracks slightly increased in length. As further loading was not possible, the load of 360 lbs./sq. ft. was maintained for about a month. The rainfall in the meantime also increased the live load, but the roof was unaffected. When the loads were removed, there were no cracks observable on the top of the slab.

Conclusion

The test of the roof has shown that the flexural strength of the concrete can be successfully used to minimize the use of the steel to a very considerable extent in the roof construction. The high test load also indicates that the type of roofing described in this paper can be used in domestic houses of more than one storey. If the flexural strength can be relied upon to a geater extent than is normally assumed, then further economy in steel is possible by giving only one beam at the centre of the rooms. Further experiments aimed at studying the flexural and compressive properties of the concrete based on locally manufactured cement are therefore being carried out, and the results will be reported separately.

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References

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