ON THE RADIO-FREQUENCY SYSTEM OF A 30 MEV MICROTRON. PART II*

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The main advantage of a microtron over other types of electron accelerators are indicated, and a general description of a 30 Mev microtron is given. Consideration of the requirements of a cavity resonator from the point of view of microtron dynamics leads to the choice of a conical line resonator with flattened cones and cylindrical surface. The resonant frequency of the resonator is calculated by the method of perturbation. The resonator was designed and constructed in the laboratory and its electrical properties measured. The results are found to be satisfactory.

Introduction

The conventional cyclotron is quite unsuitable for accelerating electrons to more than a few tens of kev., because the relativistic increase of mass causes a departure from synchronism between orbit period and the period of the radio-frequency supply. The synchrotron and the betatron can be used to produce high energy electrons for studying the charge distribution in nuclei through the scattering of fast electrons by these nuclei. These machines are relatively complicated and tend to be unreliable in operation; also, the intensity of the beam from such machines is rather low for accurate scattering experiments, due to the difficulty of extracting the beam.

The linear accelerator is very suitable for getting an intense beam of fast electrons, and with the development of pulsed microwave oscillators and amplifiers, capable of delivering megawatts of pulsed power at centimetre wavelengths, it is now possible to visualize linear electron accelerators using wave-guide techniques to provide energies approaching 1 Mev. per foot of accelerator.

Veksler[1] pointed out that when the peak voltage across the accelerating gap has certain discrete values, corresponding generally to integral multiples of the electron restmass, then a resonant acceleration which is in some ways analogous to that occurring in the cyclotron can take place without the complication of frequency modulation or varying magnetic fields. This principle is used in the microtron or the electron cyclotron which is capable of producing electrons of constant and known energy with ease of extraction of the beam.

A 4.5 Mev. microtron was built at University College, London. This microtron was found to be remarkably stable in operation and the beam was brought out of the machine by very simple means.[2] This machine uses a relatively low, steady and uniform magnetic field, and thus the construction of the magnet is simplified and there is little trouble from field inhomogenities. The R.F. system is far simpler and requires less precision machining than that in a typical linear accelerator. With the confidence gained during the operation of this small machine, the design and construction of a 30 Mev. microtron was undertaken. The author was associated with the development of the radio-frequency system of the machine.

1. General Description of the 30 Mev. Microtron

The principle of magnetic resonance on which the operation of microtron rests has been discussed in great detail by various authors [3], [4], [5] since its introduction by Veksler in 1945. The microwave resonant cavity is placed near the edge of a steady uniform magnetic field, with its axis of symmetry perpendicular to the direction of the magnetic field. The resonator is excited so that the peak voltage across the lips is slightly larger than the voltage corresponding to the rest mass of the electron. Electrons are emitted from one of the lips by field emission. The values of the magnetic field and the operation frequency of the cavity are adjusted so that the electrons which emerge from the hole with a total energy of two rest masses require a time corresponding to two cycles of the radio-frequency field to complete their first orbit. They will then make the next transit of the cavity at the appropriate phase for each electron to gain one additional rest mass of energy. Since the time needed for an electron to complete an orbit in the magnetic field is directly proportional to its total energy, the second orbit corresponds to a time interval of three cycles of radio-frequency field and the electrons under consideration once more arrive at the cavity at the correct phase for each to receive one additional rest mass of energy. In this manner the electron is subject to continued accleration moving in orbits of increasing radii, all contangential to the axis of the resonator. Thus after a transit of the accelerating field, the total energy

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* A major part of this work was carried out during the author's stay in the University College, London.

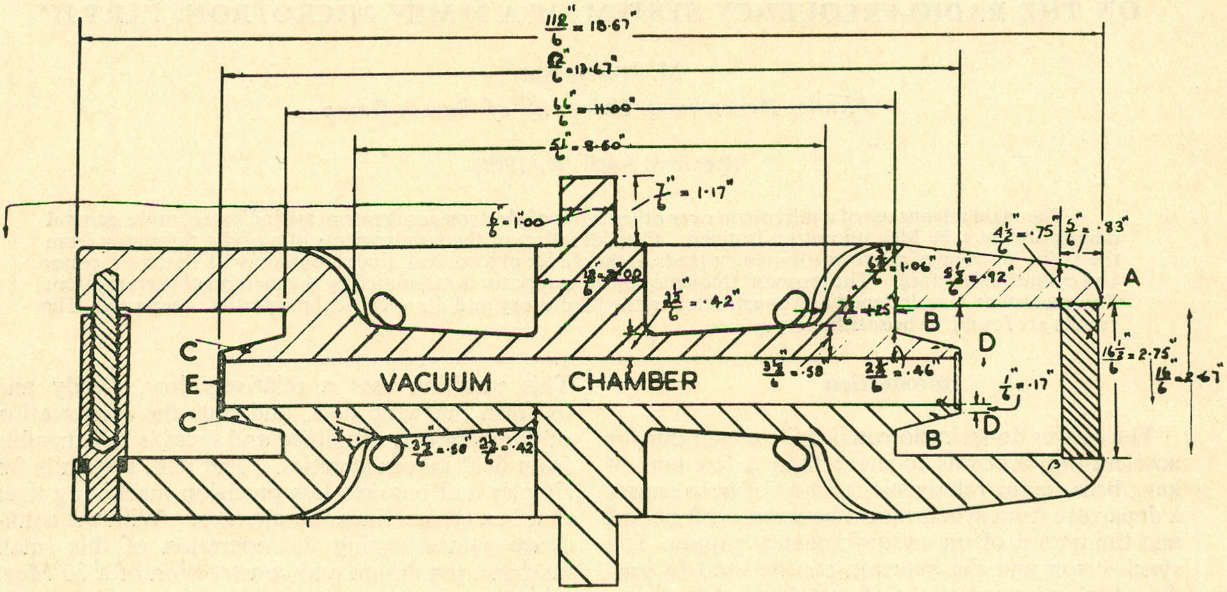

Fig. 1

of an electron is (n+1) rest masses.

The magnet has a pole diameter large enough to accommodate at least 50 orbits, with a possible upper limit of 60 orbit, in the region of uniform field. Accordingly a pole diameter of 80″ has been chosen. The air gap is 5″ so as to accommodate the larger diamension of a standard 3″× 1.5″ s-band waveguide with sufficient oom for possible auxiliary apparatus above and below the resonator. The magnetic flux density in the air gap is 1070 gauss at a wavelength of 10 cm., but the magnet is designed to provide up to 1,800 gauss to allow for possible future reduction of the wavelength. The general shape of the magnet is shown in section in Fig. 1.

Power is supplied to the magnet by a stabilized six-phase grid-controlled mercury vapour rectifier. The current supplied to the magnet is held constant to within 0.1 % during normal fluctuations of the supply voltage and the increasing resistance of the magnet coils (as their temperature increases under load). At full excitation the magnet requires about 8 amps for 1070 gauss and 18 amps for 1600 gauss. The field distribution in a one-sixth scale model magnet was measured by the electron resonance method.

The vacuum system is formed by an aluminium ring sealed to the upper and lower poles of the magnet by means of "0" ring seals. Suitable ports are provided in the ring for the entry of the R. F. waveguide and for the extraction mechanism.

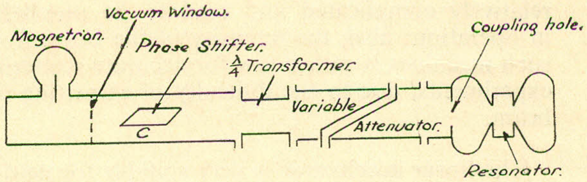

Fig. 2.—R. F. transmission system.

The radio-frequency system shown in diagram in Fig. 2 consists mainly of three parts: (*a*) the R.F. source, (*b*) the R.F. transmission system, and (c) the cavity resonator.

The source of radio frequency power is a type BM 735 pulsed magnetron capable of a rated peak output of 2000 kw. operating at 3000 MC/S. The modulator is of conventional design with an ignitron switch, and provides 3-microsecond pulses at a repetition rate of 100 pulses per second. The magnetron is coupled directly to the transmission system, the coupling being pre-adjusted during manufacture. The electrical length of the transmission line can be varied by means of a tapered dielectric line lengthener operated through a Wilson seal.

A full discussion of the R.F. transmission system and the conditions to be fulfilled for stabilization of the magnetron with resonant load has been published earlier in Part I of this communication[6] while the design and construction of the cavity resonator are described below in the next section.

2. The Cavity Resonator

From the point of view of microtron dynamics, the two major factors of the resonator are its gap parameter and outside dimensions :—

(i) The gap length must be smaller than a critical value depending on the operating mode, if electrons from a field emission source are to be accepted efficiently into phase stable orbits.

(ii) The outside dimensions of the resonator must be such that the first orbit clears the resonator.

Electrically, the resonator should have a high shunt resistance in order to minimise the power required to develop the required electric accelerating field. This requirement is in general not compatible with condition (i) outlined above. The Q-factor of the resonator should be also as high as possible to minimise the power losses in the walls.

The Gap Parameter.—Apart from the first orbit the condition necessary for resonant acceleration in the microtron is that the increase in energy obtained by an electron at each passage across the gap of the resonator shall be such as to make the electron take a whole number of periods longer for each successive orbit. Many different modes are possible. It has been shown by Heymann[2] that for a 10-cm. radio-frequency field, the best choice is the mode in which the electron gains energy corresponding to one rest mass per transit through the resonator, which therefore has a peak voltage slightly above 0.5 million volts across its lips, and the magnetic field is so adjusted that the time for each successive orbit increases by one period of the radio-frequency field. Under these conditions the gap parameter must be less than 0.81 cm. in length if phase stable operation of the microtron is desired. A gap length of 0.762 cm. was chosen for the final cavity.

The outside dimensions of the resonator and the diameter and length of the cylindrical holes through the cones are determined from consideration of clearance of the first orbit. From the electrical point of view, the hole diameter should be as small as possible and its length should be large enough to prevent radiation. But from the point of view of clearance of the beam, the hole must not be less than a minimum in its diameter and must not be greater than a maximum in its length. Hence compromises have to be made in selecting the size of the holes, and values of 0.375″ in diameter and 0.25″ in length are found to satisfy all the conditions reasonably well.

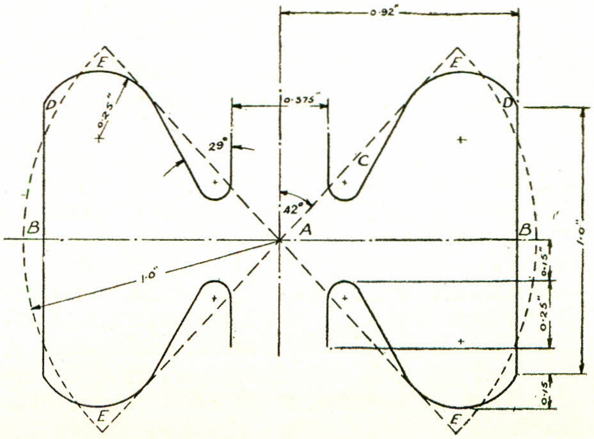

Fig. 3

Design Considerations of Shape.—Having fixed the gap length, outside size and hole parameters, the inside shape of the cavity was determined to get the highest Q and shunt resistance with the simplest possible mathematical considerations. The properties of conducting enclosures of various shapes were examined. The conical line resonator with flattened cones and cylindrical surface as shown in Fig. 3 (solid line) was found to be the most suitable for the purpose. The dotted line shows the form of the ideal conical line resonator whose electrical characteristics can be exactly calculated. The ideal cavity was modified in five regions A, B, C, D, and E to make the actual cavtiy.

The change of resonant frequency due to these modifications was calculated by the method of perturbation given by Cunlife and Mathias.[7] In actual operation the resonator had to be coupled to the waveguide conveying power from the magnetron. The coupling hole would decrease the resonant frequency of the resonator by an amount depending on its size. The change of resonant frequency due to the coupling hole was determined experimentally as it could not be calculated by simple mathematical methods.

3. Theoretical Calculation of Resonant Frequency

The resonant frequency, Q-factor and shunt resistance of cavity formed by two cones inside a perfectly conducting sphere, the apices of the cones being insulated respectively are given by

$$f = \frac{1}{4a\sqrt{\mu\varepsilon}} \qquad (1)$$

$$Q = \frac{30\pi^2}{R} \times \frac{\ln(\cot\theta_0/2)}{\ln(\cot\theta_0/2) + 0.825 \operatorname{cosec}\theta_0} \qquad (2)$$

$$R = \frac{14400\pi}{R} \times \frac{[\ln(\cot\theta_o/2)]^2}{[\ln(\cot\theta_o/2)+0.825 \operatorname{cosec}\theta_o} \quad (3)$$

where a=radius of sphere, θ_o = semi-angle of the cone, $R = 2.61 \times 10^{-7}$ f for pure copper.

In our cavity the apices of the cones are flattened and the spherical surface around the meridian plane at right angles to the cone axis is pushed inward to form a cylindrical surface. We shall treat this cavity mathematically in a way that the deformations are considered as small perturbations which change the resonant frequency slightly but have negligible effects on the field quantities. We shall calculate the change of resonant frequency by the methods of perturbation discussed below.

Methods of Perturbation.—For and two vectors A and B we have

$$\int_v B.\,\text{curl}\,A\,dv = \int A.\,\text{Curl}\,B\,dv, \text{ if } A\times ds \text{ or } B\times ds = o \quad (4)$$

Now let $e=e_o$, $k=k_o$ and $h=h_o$ before perturbation and $e=e_o+e_1$, $k=k_o+k_1$ and $h=h_o+h_1$ after pertubation. In general the incremental electric and magnetic fields e_1 and h_1 inside the perturbed volume will differ from those outside it in the cavity. Let us put $e_1=e_1'$ and $h_1=h_1'$ inside the perturbed volume which is very small in comparison with the total volume of the cavity.

By Maxwell's equation we have

$$\text{Curl } e=kh;\ \text{Curl } h=ke;\ k^2=w^2\,\mu\varepsilon \quad (5)$$

In equation (4) first let $B=h_o$ and $A=e_o+e_1$ and integrate over the total volume v of the perturbed cavity; and secondly let $B=h_o+h_1$ and $A=e_o$ and integrate over $(v-\Delta)=v_o$

Thus we get

$$(k_o+k_1)\int_{v_o} h_o(h_o+h_1)\,dv + (k_o+k_1)\int_\nabla h_o(h_o+h'_1)\,dv$$

$$= k_o\int e_o(e_o+e_1)dv + k_o\int_\nabla e_o(e_o+e_1')\,dv$$

and

$$k_o\int_{v_o} h_o(h_o+h_1)\,dv = (k_o+k_1)\int_{v_o} e^\circ(e_o+e_1)\,dv$$

Substracting and neglecting higher than the first order terms, we get

$$\frac{k_1}{k_o} = \frac{\int_\nabla e_o^2\,dv - \int_\nabla h_o^2\,dv}{\int_\nabla e_o^2\,dv + \int_{v_o} h_o^2\,dv} \quad (6)$$

Another formula given by Cunliffe and Mathias[7] is as follows:

$$\frac{k_1}{k_o} = \sqrt{\left\{\frac{1+\int_\nabla e_o^2\,dv \Big/ \int_{v_o} e_o^2 dv}{1+\int h_o^2\,dv \Big/ \int_{v_o} dv}\right\}} - 1 \quad (7)$$

The actual cavity has the form shown by the solid line in Fig. 3, while the dotted line shows the form of the theoretical conical line resonator, with $\theta_o=42°$ and radius equal to 2.54 cm., the resonant frequency f being 2943 megacycles per second. The theoretical cavity is perturbed in five regions, A, B, C, D and E to form the actual cavity. From the dimension of the cavity and the above deformations, the changes of frequency due to the perturbed regions are calculated from formulae 6 and 7, and the mean is taken. The results are tabulated in Table 1.

TABLE 1.—CHANGE OF FREQUENCY DUE TO PERTURBATION IN REGIONS A,B,C,D,E.

Region		Change of frequency by formula (6) (Mc/Sec.)	Change of frequency by formula (7) (Mc/Sec.)	Mean change (Mc/Sec.)
A	..	207.70	203.30	205.50
B	..	21.40	21.00	21.20
C	..	—54.76	—57.94	—56.35
D	..	—68.29	—68.10	—68.20
E	..	35.60	35.00	35.30

The calculated resonant frequency of the final form of cavity is 3076.36 Mc/Sec.

4. Construction and Operation of the Cavity

The body of the resonator was machined from copper, while the discs with conical poles were machined from brass. Inside surfaces of the discs and main cylindrical block were highly polished and coated with silver by evaporation in vacuum. The resonator was water-cooled through a series of holes in the body. It was decided to make the resonant frequency of the resonator nearly equal to that of the magnetron. This was done in the following way. As the magnitude of the decrease of resonant frequency of the resonator due to the coupling hole in it was not known, an experimental cavity was first constructed with a theoretical resonant frequency higher than the required value. The resonant frequency of this experimental cavity with a small coupling hole was determined experimentally. The difference between the measured value and the theoretical one was assumed to be the effect of the coupling hole. The resonant frequency was then made equal to the required value by altering the inside dimension of the cavity by an amount estimated by the method of perturbation. It might be mentioned in this connection that the magnetron was capable of tuning within 3 megacycles from its centre frequency. In the latest design the conical pieces were made by the electrolytic processes. Thus it was possible to make them reasonably thin thus making the cavity tunable within certain limits.[8] The coupling between the resonator and the waveguide was found to be extremely critical to changes in the diameter of the coupling hole. The correct diameter was found empirically by progressively increasing the hole diameter until the desired degree of coupling was obtained. The machined hollow in the body of the cavity, between the cavity and the waveguide, illustrated at 'D' in the Fig. 4 was provided so as to avoid the necessity of feeding the resonator through a long coupling hole operating essentially in a cut-off mode. The absence of this groove would have necessitated the use of a coupling hole with a much larger diameter, resulting in a correspondingly larger disturbance of the field in the resonator. The various views and dimensions of the cavity are given in Figs. 3, 5, and 6.

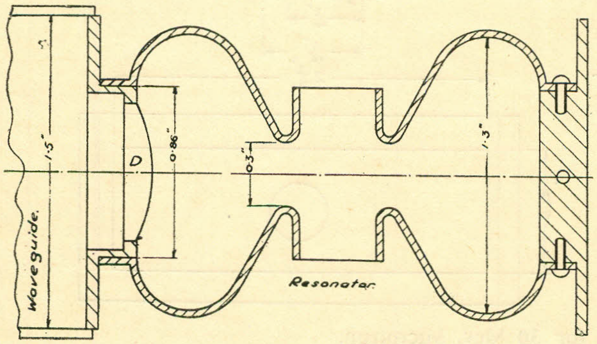

Fig. 4—Coupling of resonator to waveguide

5. Experimental Measurements

Frequency measurements were made by energizing the cavity from a highly stabilized tunable 10 cm. band Klystron (CV35), and measuring the relative response of the cavity over a range of frequencies including the resonant frequency, using extremely loose coupling.

The Q of the resonator and its circle diagram were obtained by measuring the impedance looking into the cavity over a similar range of frequencies with the standing wave method.[9] Rough and

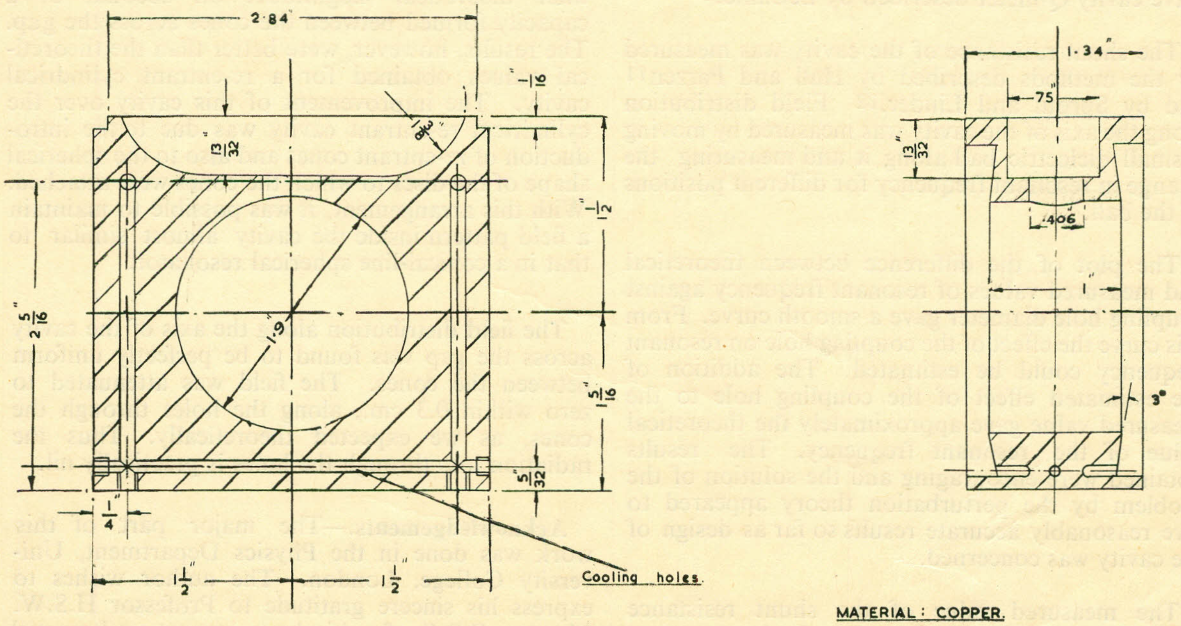

Fig. 5.—Cavity of resonator for 30 Mev. microtron.

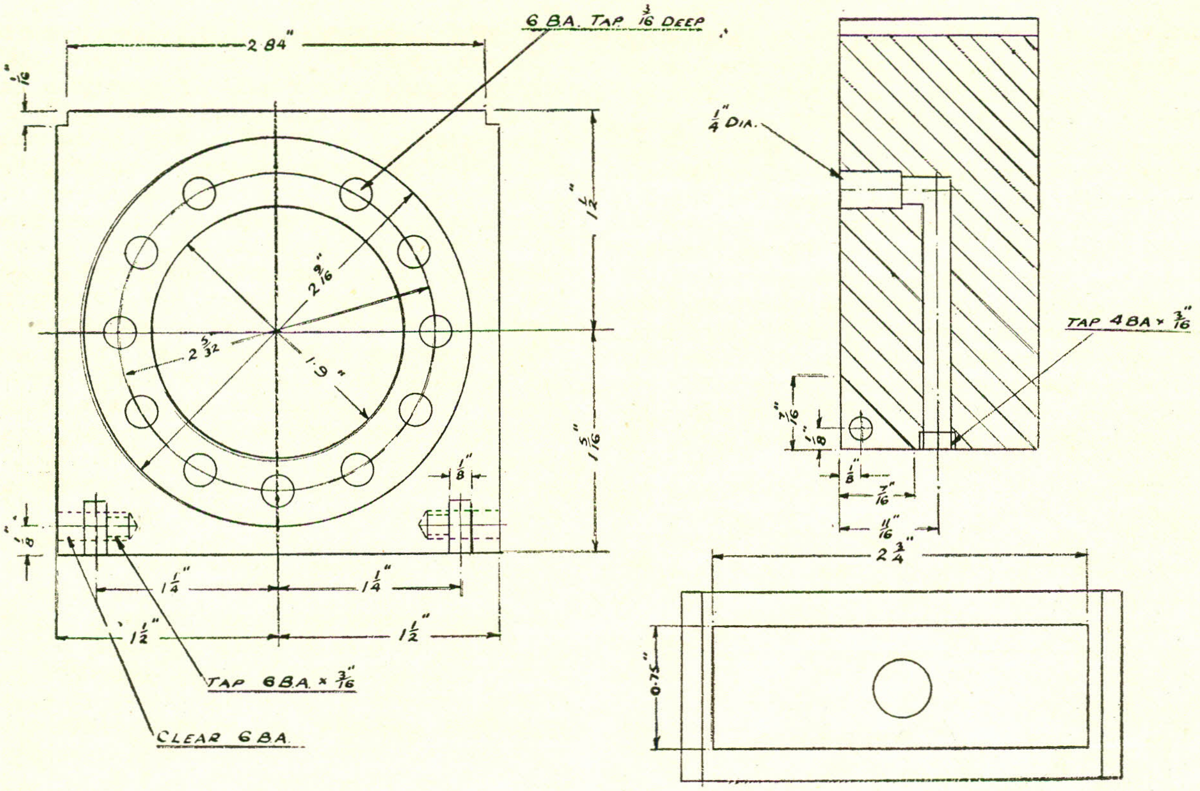

Fig. 6.—Cavity resonator for 30 Mev. Microtron.

quick estimation of the Q during experimental development of the cavity was made by a microwave cavity Q-meter described by LeCaine.[10]

The shunt resistance of the cavity was measured by the methods described by Hall and Parzen[11] and by Sproul and Linder.[12] Field distribution along the axis of the cavity was measured by moving a small dielectric ball along it and measuring the change in resonant frequency for different positions of the ball.

The plot of the difference between theoretical and measured values of resonant frequency against coupling hole diameter gave a smooth curve. From this curve the effect of the coupling hole on resonant frequency could be estimated. The addition of the estimated effect of the coupling hole to the measured value gave approximately the theoretical value of the resonant frequency. The results obtained were encouraging and the solution of the problem by the perturbation theory appeared to give reasonably accurate results so far as design of the cavity was concerned.

The measured value of the shunt resistance (0.71 megohms) for the silvered cavity however was approximately 50% lower than the theoretical value. The values of both Q and shunt resistance of the actual cavity constructed were lower than their theoretical magnitudes on account of a capacity formed between the cones across the gap. The results, however, were better than the theoretical values obtained for a re-entrant cylindrical cavity. The improvement of this cavity over the cylindrical re-entrant cavity was due to the introduction of re-entrant cones and also to the spherical shape of the discs to which the cones were attached. With this arrangement, it was possible to maintain a field pattern inside the cavity almost similar to that in a conical-line spherical resonator.

The field distribution along the axis of the cavity across the gap was found to be perfectly uniform between the cones. The field was attenuated to zero within 0.3 cm., along the holes through the cones, as we expected theoretically. Thus the radiation loss through the holes is practically nil.

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References

1. V. Veksler, J. Phys. (U.S.S.R.), **9**, (1945).
2. F.F. Heymann, Ph. D. Thesis, London University, 1953.
3. Readhead et el., Canadian J. Research, **A 28,** 73 (1950).
4. J. Itoh and D. Kobayshi, Collected Papers Fac. Sci., Osaka Univ., **BII** (1950).
5. C. Henderson et el., Proc. Phys. Soc. (London), **66B,** 49 (1953).
6. M.I. Ali, Pakistan J. Sci. Ind. Research, **1,** 16 (1958).
7. A Cunlife and L.E.S. Mathias, Proc. Inst. Elec. Engrs. (London), Pt. III., **97,** 367 (1950),
8. F.F. Heymann, private communication (1957).
9. Montgomery, *Technique of Microwave Measurements* (1948).
10. H. LeCaine, Proc. I.R.E., **40**, 155 (1952)
11. G. L. Hall and Pargen, Proc. I.R.E., **41,** 1760 (1953).
12. Sproul and Linder, Proc. I.R.E., **40**, 305 (1946).