PROTON-PROTON SCATTERING AT MEDIUM-HIGH ENERGIES

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Scattering experiments between nucleons constitute some of the most fundamental problems in nuclear physics and give us information about the nature of the force acting between them. In the case of proton-proton scattering, the Pauli Exclusion Principle rules out the triplet states for waves of even angular momentum (including $l = 0$) and singlet state for those of odd angular momentum. As the nuclear force is a short-range one, partial waves of $l > 2$ cannot enter the field of nuclear force, for the energy range considered here, and may thus be excluded. Even then, it has been found by recent experiments that the nuclear potential is only effective in the ^1S-state and possibly in the ^3P-state. The interference between the Coulomb and the nuclear force gives a characteristic minimum in the angular distribution and this determines the sign as well as magnitude of the nuclear phase shift which gives quantitative information on the parameters of the nuclear potential.

Breit and his collaborators[1] showed, for the first time, that the deviation of the scattering cross-section from Mott scattering is due to nuclear effects. They explained the observed discrepancy by replacing the Coulomb phase shift, σ_l, by a modified phase shift, $\sigma_l + \delta_l$, in the asymptotic solution of the Schrodinger equation for the (p-p) system. The quantity δ_l which is called the nuclear phase shift, is independent of angle and can explain the entire angular distribution at a given energy. The phase shift was related by the authors to a nuclear potential of a rectangular shape whose range and depth was $\sim 2.8 \times 10^{-13}$ cm. and ~ 10 MEV respectively. As the energy range they considered extended only upto 2.4 MEV, they considered the nuclear effect in the s-state only.

Following this pioneer work, a number of experiments were made on (p-p) scattering and more detailed knowledge of the nucleon potential was sought. Landau and Smorodinsky[8] predicted a simple functional form for the variation of the nuclear phase shift with energy under very general assumptions about the nuclear potentials. But they did not succeed in giving a rigorous mathematical derivation of the formula. Schwinger,[10] at last, related the formula to the quantum mechanical properties of the system and explained Landau and Smorodinsky's approach by a variational method. He showed that the nuclear phase shifts can be related to a set of variational parameters which, in turn, will predict the differences in the scattering properties of a short-tailed and a long-tailed well.

We shall consider here this Landau-Smorodinsky-Schwinger approach to scattering problems.[7] Under the assumptions of a short-range interaction, a certain function, K, of the nuclear phase shift and the velocity 'v' of the protons is defined as

$$K = \frac{\pi \cot \delta_0}{e^{2\pi\eta} - 1} + h(\eta) \quad \ldots\ldots\ldots \quad (1)$$

where δ_0 = s-wave phase shift

$$h(\eta) = -\ln \eta - 0{\cdot}5772 \ldots\ldots + \eta^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+\eta^2)}$$

$$\text{and } (\eta) = \frac{e^2}{hv}$$

The first term in the right hand side of the equation is analogous to the familiar term $k \cot \delta$ in the neutron-proton case, multiplied by the Coloumb penetration factor. The second term is due to the infinite range of the Coloumb force and is a slowly varying function of energy.

The above function, K, is related to the f-function of Breit, Condon and Present[1] and that of Bethe[3] by the relation :—

$$K = \tfrac{1}{2} f - 0.15443$$

$$\& \ f_{Bethe} = f + 0{\cdot}84457 \quad \ldots\ldots\ldots \quad (2)$$

Now K allows a power series expansion in the energy giving :—

$$K = R(-a^{-1} + \tfrac{1}{2} r_0 k^2 - P r^3_0 k^4 + Q r^5_0 k^6 - \ldots) \quad \ldots (3)$$

where R = Bohr radius of a proton bound to a fixed unit charge

and k = wave number of relative motion.

The first two co-efficients in the power series have a physical meaning. 'a' is equivalent to the Fermi scattering length,[5] evaluated at zero energy ; 'r_0' is called the effective range (of the potential well), though it is dependent both on the actual depth and the range of a particular well. If we neglect other terms in the power series, K is a linear function of energy. The two parameters 'a' and 'r_0' can thus be fitted to *any* well shape which has two adjustable constants — well depth and range. The other co-efficients (P, Q, ...etc.) of equation (3) are a measure of the deviation of the K vs. energy graph from linearity and determine the shape of the nuclear potential. It may be mentioned here that Bethe[3] has also developed a similar power series in terms of his f-function from considerations of the 'effective range theory'. One can, therefore, use either the f-function or the function K in analysing the phase shifts from proton-proton scattering data.

Application to experimental results : As has already been pointed out, experiments on (p-p) scattering below 10 MEV could hardly detect any p- or d-wave effects and experiments even upto 32 MEV produced scattering which was predominantly s-wave only.[4] Let us assume that scattering occurs in the s-state and the p-state only, upto an energy of 32 MEV and tensor forces are non-existent. Then, as the s-wave and the p-wave contributions are incoherent and the p-wave contribution is zero at 90°, a cross-section at this angle, in the centre-of-mass system, will give the s-wave phase shift exactly even if p-wave scattering is present. This phase shift will give a value of K from equation (1) and the value can be plotted against k^2 (which is a function of energy). Jackson and Blatt[7] have analysed this 90°-scattering data for values of E upto 3.5 MEV, for which region accurate values of scattering cross-section were then available. They found that all the scattering data could be related to a shape-independent formula and hence they could not discriminate between different potential well shapes. Recently Yovits et al.[11] have summarised information on low and intermediate-energy scattering. Their results show a long-tailed Yukawa or exponential potential and in the former case, the corresponding meson mass is 333 m_e.

In view of the importance of the above results in determining the shape of the nuclear potential, all the available data below 40 MEV has been collected by the author. A fresh analysis on the lines of above theory was made and the value of K obtained from each result was plotted against k^2. Table I gives all the values thus obtained :

TABLE I.—PUBLISHED DATA ON (P-P) SCATTERING

Source		E(MEV)	k^2($\times 10^{24}$cm^{-2})	δ_a	K
RKT	..	.1675	.213	5.78±0.35	3.79±0.16
,,	..	.2002	.241	6.80±0.32	3.82±0.14
,,	..	.2259	.272	7.87±0.30	3.87±0.12
,,	..	.2495	.301	9.03±0.30	3.83±0.11
,,	..	.2753	.332	10.06±0.28	3.82±0.09
,,	..	.2983	.359	10.96±0.26	3.91±0.09
,,	..	.3214	.387	11.82±0.30	3.93±0.15
HHT	..	.670	.807	24.68±0.40	4.00±0.10
,,	..	.776	.935	27.12±0.40	4.12±0.08
,,	..	.867	1.045	29.32±0.40	4.17±0.07
HKPP	..	.860	1.036	29.28±0.40	4.15±0.03
,,	..	1.200	1.446	35.94±0.40	4.32±0.03
,,	..	1.390	1.675	38.76±0.40	4.41±0.03
,,	..	1.830	2.206	44.02±0.40	4.59±0.02

TABLE 1 (*Contd.*)

Source		E(Mev)	$k^2(x10^{24}cm^{-2})$	δ_a	K
HKPP	..	2.105	2.537	46.18±0.40	4.72±0.03
,,	..	2.392	2.883	48.08±0.40	4.85±0.03
BFLSW	..	2.42	2.917	48.24±0.50	4.86±0.05
,,	..	3.04	3.664	50.95±0.50	5.15±0.06
,,	..	3.27	3.941	51.89±0.50	5.24±0.06
,,	..	3.53	4.254	52.58±0.50	5.36±0.07
RWH	..	2.42	2.917	47.91±0.40	4.90±0.05
,,	..	3.04	3.664	50.80±0.30	5.17±0.05
,,	..	3.28	3.953	51.77±0.40	5.26±0.06
,,	..	3.53	4.254	52.20±0.30	5.41±0.06
MP	..	4.2	5.06	52.70±2.00	5.83±0.30
M	..	4.94±.04	5.95± .06	54.70±1.00	6.02±0.18
WC	..	8.00±.10	9.64± .12	52.70±2.00	8.00±0.40
WLRWS	..	14.5 ±.70	17.5 ±0.8	52.20±3.50	10.78±0.80
R	..	3.44	4.145	52.46±0.60	5.34±0.06
,,	..	6.85	8.254	55.82±0.60	6.86±0.12
,,	..	7.57	9.050	55.78±0.60	7.18±0.13
Mather	..	5.07	6.109	54.50±0.60	6.11±0.14
ZK	..	5.86	7.061	55.60±0.30	5.42±0.05
FW	..	12.4	14.701	53.30±0.07	9.68±0.20
AABR	..	9.7 ±.15	11.7 ± .18	57.8 ±1.20	7.68±0.20
PF	..	29.4	36.427	K is given directly in the published paper.	15.84±0.39
CJR	..	31.8	38.319	,,	16.76±0.25
Cork	..	18.8	22.654	,,	11.67±0.27
,,	..	21.9	26.390	,,	12.91±0.27
,,	..	25.2	30.366	,,	14.65±0.33
,,	..	25.45	30.667	,,	14.84±.27
,,	..	31.45	37.897	,,	16.73±.30
KKK	..	5.77	6.953	54.96±0.16	6.48±.025
WMF	..	4.2	5.061	53.80±0.08	5.69±.009
MV	..	18.3	22.052	52.8 ±0.08	11.825±.20
Hossain	..	9.5 ±.02	11.4 ±.02	53.9 ±1.5	8.56±.24
CH	..	9.7 ±.05	11.6 ±.05	56.5 ±0.10	7.94±.10
YW	..	18.3 ±.24	21.8 ±.24	53.2 ±0.90	11.40±.20

References for the abbreviations in Table I

1. RKT — To be found in the Table given by Jackson and Blatt.
2. HHT — To be found in the Table given by Jackson and Blatt.
3. HKPP — To be found in the Table given by Jackson and Blatt.
4. BFLSW — To be found in the Table given by Jackson and Blatt.
5. RWH — To be found in the Table given by Jackson and Blatt.
6. MP — To be found in the Table given by Jackson and Blatt.
7. M — To be found in the Table given by Jackson and Blatt.
8. WC — To be found in the Table given by Jackson and Blatt.
9. WLRWS — To be found in the Table given by Jackson and Blatt.
10. R — Rouvina, Phys. Rev., **81,** 593 (1951).
11. Mather — Mather, Phys. Rev., **82,** 133 (1951).
12. ZK — Zimmerman and Kruger, Phys. Rev., **83,** 218A (1951).
13. FW — Faris and Wright, Phys. Rev., **79,** 577 (1950).
14. AABR — Allred, Armstrong, Bondelid and Rosen, Phys. Rev., **88,** 423 (1952).
15. CJR — Cook, Johnston and Richman, Phys. Rev., **79,** 71 (1950).
16. C — Cork, Phys. Rev., **80,** 321 (1950).
17. KKK — Kerman, Kreger and Kruger, Phys. Rev., **89,** 908A (1953).
18. WMF — Worthington, McGruer and Findley, Phys. Rev., **90,** 899 (1953).
19. MV — Martin and Verlet, Phys. Rev., **89,** 519 (1953).
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21. CH — Cork and Hartsough, Phys. Rev., **94,** 1300 (1954).
22. YH — Yntema and White, Phys. Rev., **95,** 1226 (1954).

A least square method was used to obtain the best value of the parameters, a, r_0, P, in equation (3). The equation found for the best fit is

$$K = 3.6856 + 0.4158(5)k^2 - 0.001942k^4 \ldots\ldots (4)$$

This gives:

$$a = -7.818 \times 10^{-13} \text{ cm.}$$
$$r_0 = 2.886 \times 10^{-13} \text{ cm.}$$
$$P = +0.028$$

Figures 1 and 2 show that while the line of best fit is indistinguishable from a straight line upto an energy of about 5 MEV, appreciable departure from P = 0 line is indicated by the same curve beyond 5 MEV. The value of P clearly indicates that the effective potential is a long-tailed one and the potential well falls between the exponential and the Yukawa type. If we assume a Yukawa well, which is the only one with a theoretical basis, the corresponding meson mass is 318 m_e. This is in agreement with the value obtained by Breit and Hatcher[2] when they express their f-function as a quadratic series in E, but is lower than the masses obtained by Yovits et al.[11] and Hoisington et al.[6] The difference of mass between this Yukawa particle and π-meson, whose mass is 273 m_e, is much more than any relativistic correction, or a correction due to a cut-off in the meson potential. Breit and Hatcher,[2]

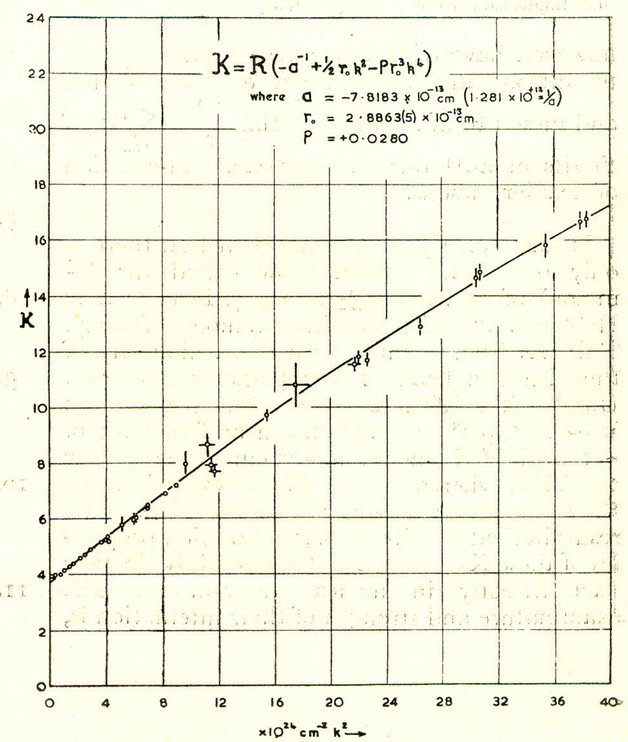

Fig. 1.

Experimental values of K plotted vs. k2 for all published data on (p-p) scattering below 40 MEV. The least square fit to these experimental points is also given, showing that the best curve is quadratic.

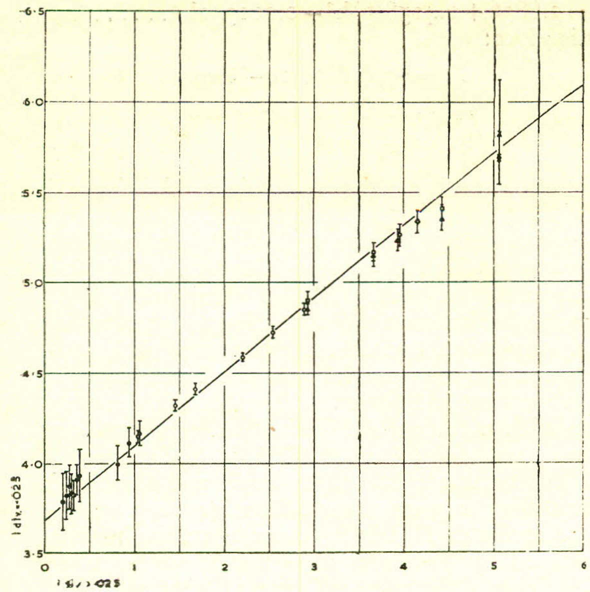

Fig. 2

A detailed and magnified plot of the experimental points at energies 5 MEV, with the least square line of figure 1. It is seen that for such low energies, the curve is partically indistinguishable from a straight line.

however, have tried to explain this discrepancy by introducing a correction to the Coloumb and meson term in the potential, $\frac{\epsilon^2}{r} - \frac{C e^{-\gamma/a}}{\gamma/a}$, of Yovits et al.[11] due to short-range interaction of heavier mesons.

It has recently been established that there is only one heavy meson (now called the K-meson) of mass $965 \pm 10\ m_e$ (Ritson et al.,[9] Hebb et al.,[6]) which has different modes of decay. This has been possible with the availability of the K-meson beam at the Berkeley Bevatron, U.S.A. The lifetime of these unified K-mesons is $\sim 1 \times 10^{-8}$ sec. and the mean free path of $\sim$100 MEV K^--mesons is $\sim$25 cm. in the nuclear emulsion, whereas that of K^+-mesons of about the same energy is $\sim$95 cm. in emulsion. The result indicates quite strong interaction properties of these K-mesons and they possibly do take part directly in nuclear interactions. The exact nature and strength of their interaction is, however, still uncertain. It is therefore understandable that the meson mass obtained in proton-proton experiments (assuming Yukawa potential) will be larger than the π-meson, as it is the effective mass due to contributions from both π- & K-mesons.

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