SCATTERING OF HIGH ENERGY ELECTRONS IN NUCLEAR EMULSION

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During the last few years the photographic emulsion technique has been applied to a wide variety of problems concerning nuclear studies. As far as the author is aware there have been no previously published investigations of electron-electron scattering using this technique. However, soon after the present work was started some work of Barkas et al.[1] from Berkeley using this technique for the study of positron electron scattering was published. This is perhaps surprising since the study of delta rays (*i.e.*, electrons with small fractional energy loss) has in recent years been applied frequently to the identification of particles in photographic emulsion.

The photographic emulsion methods are useful because they provide permanent records of the tracks of fast electrons and all the details of collision processes can be studied carefully at leisure. Since the electron density in the material of the emulsion is comparatively well-known it is possible to obtain unambiguous results of the collision cross-sections in this case. It was, therefore, decided to use the emulsion technique to study electron-electron scattering.

The method enables one to obtain results in the case of collisions involving small fractional energy losses of the incident electrons. It would be interesting also to obtain results for larger fractional energy losses but such events are rare and, therefore, the stati tical uncertainties in the observed values are high. Nevertheless, the previous work on electron-electron scattering is so meagre and confined to so few incident-electron energies that it seemed worthwhile to carry out the present investigations inspite of the limitation to rather small energy losses. A short note on the present work was published earlier.[2]

Theory

The most rigorous treatment of the problem of electron-electron collision is due to Moller.[3] His formula for the cross-section of the scattering per electron between angles $\theta^\star$ and $\theta^\star + d\theta^\star$ (in the centre of mass system) is

$$\sigma(\theta^\star)\, d\theta^\star = \frac{(\gamma+1)\pi r_0^2 \operatorname{Sin}\theta^\star d\theta^\star}{\gamma^2\beta^4}\Big[\operatorname{Cosec}^4 \theta^\star/_2 + \operatorname{Sec}^4 \theta^\star/_2 - \operatorname{Cosec}^2 \theta^\star/_2 \operatorname{Sec}^2 \theta^\star/_2 + \left(\frac{\gamma-1}{\gamma}\right)^2 (1 + 4 \operatorname{Cosec}^4 \theta^\star)\Big] \quad \ldots\ldots(1)$$

where r_0 is the classical electron radius, $\beta = v/_c$ and $\Upsilon = \frac{1}{\sqrt{1-\beta^2}}$ = total energy (kinetic + rest mass) of the incident electron in units of m_0c^2, the rest mass energy of the electron. The first two terms within the bracket correspond to the relativistic Rutherford formula. The third term is the quantum mechanical exchange term. The inclusion of this term in the relativistic Rutherford formula gives the relativistic Mott[4] or the Kar and Basu[5] formula. The fourth term accounts for the retardation effect and its inclusion in the relativistic Mott or the Kar and Basu formula gives the Moller formula. This term, however, vanishes in the non-relativistic limit $\Upsilon \to 1$.

The formula, according to Moller, may be expressed in terms of a new variable A, the ratio of the kinetic energy of the knock-on electron to the kinetic energy of the incident electron. By a simple transformation the Moller formula (1) may be written as

$$\sigma(A)dA = \frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(1-A)^2} - \frac{3}{A(1-A)} + \left(\frac{\gamma-1}{\gamma}\right)^2\left\{1 + \frac{1}{A(1-A)}\right\}\right] dA \ldots\ldots(2)$$

The relativistic Rutherford formula in the same notation is

$$\sigma(A)dA = \frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(1-A)^2} - \frac{2}{A(1-A)}\right] dA \ldots\ldots\ldots(3)$$

The relativistic Mott or the Kar and Basu formula is

$$\sigma(A)dA = \frac{2\pi r_0^2}{\beta^2(\gamma-1)}\left[\frac{1}{A^2(1-A)^2} - \frac{3}{A(1-A)}\right] dA \ldots\ldots(4)$$

Technique and Measurements

(*a*) *Exposure of plates :* Ilford G5 plates of dimensions 2″×2″ and emulsion thickness 200 microns were exposed to electrons accelerated to high energies by a 20-MEV synchrotron at the University College, London. The plates were then processed by the so-called "temperature development" method of Dilworth et al.[6]

*The experimental part of the work described here was done at the University College, London, while holding an 1851 Exhibition Scholarship.

(*b*) *Measurements :* Measurements of the plates were carried out using a Beck binocular microscope. A 2 mm (~ 80 x) oil immersion objective was used in conjunction with 15 x compensated eyepieces fitted with a calibrated scale (1 division = 0.635μ) and a protractor provided with a vernier (1 division = 10 minutes).

The plates were mounted on the mechanical stage of the microscope in such a way that the tracks were approximately parallel to the y-axis of the stage motion. The tracks, which started right from the surface of the emulsion were followed by advancing the stage of the microscope by means of the screws, and the length traversed by the electron was read on the vernier scale of the stage. 90 cm. of track were scanned at each incident energy *viz.* 6.5, 11.5 and 18.7 MEV.

Whenever an electron-electron collision was observed, both the range of the knock-on electron and the angle between its direction and that of the incident electron were measured, wherever possible. These measurements were made by means of the calibrated scale and the protractor, respectively. For very low energy knock-ons the angle became very difficult to measure because of strong nuclear scattering. Therefore, the range was the principal means of determining the energy of the knock-on electron upto about 0.5 MEV. It was, however, observed that most of the recoil electrons with energies greater than 0.2 MEV went out of the emulsion. For the higher energies, therefore, we had to depend on the angle measurements only. Very good agreements were observed when both measurements were possible on individual tracks.

The true length, L, of the recoil track is given by

$$L^2 = l^2 (1 + \tan^2\beta) \quad \ldots\ldots (5)$$

where l is the projected length of the recoil track and β, the angle of dip. The energy of the recoil electron was then found out from the range-energy relation of electrons for photographic emulsion, as has been measured by Zajac and Ross.[7]

The true angle, Φ, which the recoil electron makes with the incident electron is given by

$$\cos \Phi = \cos \alpha \cos \beta \quad \ldots\ldots (6)$$

where α is the angle of ejection projected on the plane of emulsion containing the incident electron and β the dip angle. The energy, Q, of the recoil electron was then calculated from the angle-energy relation

$$Q = \frac{E \cos^2 \Phi}{1 + \dfrac{E \sin^2 \Phi}{2m_0c^2}} \quad \ldots\ldots (7)$$

where E is the energy of the incident electron and m_0c^2 the rest mass energy of the electron.

Results

(*a*) *Calculation of theoretical cross-section :* Calculations of the theoretical cross-sections have been carried out by integrating the differential cross-sections given by equations 2, 3 and 4. After integrations are carried out, the three equations become

Rutherford:

$$\int \sigma(A)\, dA = \frac{2\pi r_0^2}{\beta^2(\gamma - 1)} \left[-\frac{1}{A} + \frac{1}{1 - A} \right]_{A_{min}}^{A_{max}} \quad \ldots\ldots (8)$$

Mott or Kar and Basu:

$$\int \sigma(A)\, dA = \frac{2\pi r_0^2}{\beta^2(\gamma - 1)} \left[-\frac{1}{A} + \frac{1}{1 - A} - \log_e A + \log_e (1 - A) \right]_{A_{min}}^{A_{max}} \quad \ldots\ldots (9)$$

Moller

$$\int \sigma(A)\, dA = \frac{2\pi r_0^2}{\beta^2(\gamma - 1)} \left[-\frac{1}{A} + \frac{1}{1 - A} - \log_e A \left\{ 1 - \left(\frac{\gamma - 1}{\gamma}\right)^2 \right\} + \log_e (1 - A) \left\{ 1 - \left(\frac{\gamma - 1}{\gamma}\right)^2 \right\} + \left(\frac{\gamma - 1}{\gamma}\right)^2 A \right]_{A_{min}}^{A_{max}} \quad \ldots\ldots (10)$$

To obtain the total expected number of events in any knock-on energy interval, say 0.2-0.5 MEV, the results of the integrated equations 8, 9 and 10 were multiplied by the total number of electrons in 90 cm. of the emulsion that lie across the incident electron path. From the composition of Ilford G5 emulsion, as supplied by the Ilford Ltd., the number of electrons in 1 cc. of emulsion was calculated to be 1.07×10^{24}.

(*b*) *Loss correction and Results :* It appears likely that some tracks scattered sharply down into the emulsion will have been missed in the measurements. The knock-on electrons which should be uniformly distributed in Azimuth

TABLE

E_I (MEV)	E_R (MEV)	N	Theoretical estimate of no. of events		
			Rutherford (relativistic)	Mott or Kar & Basu	Möller
6.5	0.03 – 0.06	424±23	419	419	419
	0.06 – 0.10	158±13	164	164	164
	0.10 – 0.20	117±11	122	122	122
	0.20 – 0.50	71± 9	74	74	74
	0.50 – 1.00	22± 5.5	24.5	22	25
	1.00 – 2.00	10± 3.5	13	10	14
	2.00 – 3.25	6± 2.5	8	5	9
11.5	0.03 – 0.06	448±27	408	408	408
	0.06 – 0.10	187±14	162	162	162
	0.10 – 0.20	121±11	120	120	120
	0.20 – 0.50	64± 8	72	72	72
	0.50 – 1.00	27± 5.5	24	24	24
	1.00 – 2.00	13± 3.5	12.5	11	13
	2.00 – 5.75	9± 3	10	7	11
18.7	0.03 – 0.06	394±24	401	401	401
	0.06 – 0.10	145±13	160	160	160
	0.10 – 0.20	102±10.5	117	117	117
	0.20 – 0.50	63± 8	67	67	67
	0.50 – 1.00	22± 4.5	24	24	24
	1.00 – 2.00	9± 3	12.5	11	13
	2.00 – 5.00	5± 2.5	6	4.8	6.5
	5.00 – 9.35	1± 1	1.9	1.5	2.2

E_I = incident energy ; E_R = recoil energy ;
N = number of observed events, including loss correction.

about the incident direction, actually showed a marked deficiency of recoils corresponding to ejection at a large angle to the plane of the surface of the emulsion, as might have been expected from the difficulty of observation under the conditions, so that it was necessary to make a loss correction. From the departure of the distribution in Azimuthal angle from uniformity, it was estimated that this loss correction was 11%, 12% and 16.5% for incident electron energies of 6.5, 11.5 and 18.7 MEV, respectively.

The corrected numbers of recoils in the different ranges of energy are shown in the table.

(*c*) *Discussion :* Comparison of equations 8, 9 and 10 shows that, in the region of A (*i.e.* $\frac{E_R}{E_I}$) less than 0.1, the three equations are indistinguishable, the contribution from the log terms being negligibly small. In this region, the observations are seen to be in good agreement with each of the three theoretical expressions. In the region of A greater than 0.1, the three theories differ appreciably in their predictions, but unfortunately in this region the observed numbers are insufficient to make any significant comparison possible between them. In order to obtain significant differentiation, one would need to scan many times more tracks than has at present been done. This will require a great deal of scanning effort.

References

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